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JUNIOR
HIGH SCHOOL MATHEMATICS

FIRST BOOK

BY

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PREFACE

This is the first of a series of two books in mathematics for the seventh and eighth grades. These books assume that the pupil has mastered the work in arithmetic usually given in the first six grades. They continue the work in arithmetic by drill to obtain speed and accuracy, a study of percentage and its applications in ordinary business and in other everyday affairs, and a study of mensuration. They extend the mathematical content of the course of the seventh and eighth grades by including those parts of elementary algebra and geometry that are adapted to the abilities of the pupils of these grades. This extension is made possible by the omission of the more difficult and technical applications of arithmetic found in the traditional course. It is believed that these books contain enough of algebra and geometry so that a year may be gained in the later course in the high school.

Algebra is approached through the formula which is its most practical aspect for the beginning pupil. Throughout the course the pupil is given practice in stating rules as formulas and formulas as rules until the formula comes to be a natural expression for mathematical truths. Other algebraic notions included are the equation, negative numbers, and the graph. Common geometric notions — angle, perpendicular, triangle, polygon — are introduced and used in constructions and various problems in mensuration. Many of the important theorems of elementary geometry are developed from observation and construction and are used in applied problems. It is believed that the familiarity thus obtained with geometric truths will appreciably shorten

the time needed for formal geometry later in the course. In general, after algebraic and geometric notions have been introduced they are used as often as possible so as to unify the three types of work.

This book for the seventh grade contains (1) much drill in the fundamental operations of arithmetic; (2) practice in the interpretation of problems; (3) exercises in the use of the literal notation in interpreting and evaluating formulas; (4) a study of percentage and its applications to common life as well as to business problems; (5) the study of a considerable number of geometric notions, theorems, and constructions without demonstration.

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JUNIOR HIGH SCHOOL MATHEMATICS

FIRST BOOK

CHAPTER I

SOLUTION OF PROBLEMS. REVIEW OF FUNDAMENTAL OPERATIONS

1. Directions for solving problems. The following suggestions should be kept in mind when solving a problem:

1. Read the problem carefully.
2. Decide what facts the problem tells you.
3. Decide what things you need to find.
4. Think the processes by which the things needed may be found.
5. Estimate the result.
6. Perform *accurately* the processes decided upon.

In writing the solution use a form similar to the one at the bottom of the next page. On the left-hand side of the paper write the analysis or explanation of the problem. On the right-hand side write the computation, if any written computation is necessary.

EXAMPLE. A grocer buys a barrel of apples for \$3.60. He sells them at 70¢ a peck. The barrel holds $2\frac{1}{4}$ bushels. One and one-half pecks of the apples are unfit to sell. *How much does* he make on the barrel of apples?

Thought process in solving. The problem tells us that

1. The barrel of apples costs \$3.60.
2. The barrel holds $2\frac{1}{4}$ bushels.
3. One and one-half pecks of the apples are unfit to sell.
4. The grocer sells the others at 70¢ a peck.

We need to find out:

1. How many he sells.
2. How much he receives for them.
3. How much he makes on the barrel of apples.

To find out how many he sells we subtract the amount unfit to sell from the amount bought.

To find how much he receives for them we multiply the price per peck by the number of pecks sold.

To find the profit we subtract the cost from the selling price.

We estimate that:

1. He sold a little less than 2 bu.
2. He received a little more than \$5 for them.
3. He made about \$1.50 on them.

Form of solution to be written	**Computation**
$2\frac{1}{4}$ bu. $=9$ pk.	$\begin{array}{r} .70 \\ \underline{7\frac{1}{2}} \end{array}$
9 pk. $-1\frac{1}{2}$ pk. $=7\frac{1}{2}$ pk.	$\begin{array}{r} 490 \\ \underline{35} \end{array}$
$7\frac{1}{2}\times\$.70=\$5.25$.	$\begin{array}{r} 5.25 \\ \underline{3.60} \end{array}$
$\$5.25-\$3.60=\$1.65$.	1.65

Pupils should write the computation only when necessary.

Exercise 1

1. If an American flag is 4 ft. wide, how wide is each stripe?

2. One side of a lot measures 198 ft. How many posts will be required for a fence along this side if they are set a rod apart?

3. A certain varnish stain is sold in pint cans at 32¢ a can or in quart cans at 55¢ a can. One pint will stain 25 sq. ft. How many cans of stain must be bought to stain a floor 16 ft. wide and 20 ft. long? How much will it cost?

4. John's garden is 21 ft. wide and 41 ft. long. It is to be planted in popcorn in rows 2 ft. 6 in. apart. The hills are to be 2 ft. apart in the rows. How many hills will he have if the outer rows are 6 in. from the edge of the garden?

5. A boy was given 175 carnations, some red and some pink. He was told to tie the red ones in bunches of 6 each and sell them for 30¢ a bunch. He counted the pink ones and found that there were only 19. He sold all the red ones. How much did he get for them?

6. A committee is appointed to buy class colors for the 7B grade. Each pupil is to have 9 in. of each of two colors of ribbon. There are 24 pupils in the grade. The ribbon is to cost 18¢ a yard. How much must the committee collect from each pupil to pay for the ribbon?

7. Two boards are to be screwed together. One of them is $\frac{7}{8}$ of an inch thick and the other is $\frac{1}{2}$ of an inch thick. How long a screw will it take to reach through the half-inch board and half way through the seven-eighths-inch board? How long must the screw be to reach through the seven-eighths-inch board and $\frac{3}{4}$ of the way through the half-inch board?

8. A boy has $180 to spend for a pony and a cart. He plans to spend twice as much for the pony as for the cart. How much does he spend for each?

9. A board 8 ft. long is to be cut into two pieces so that one piece will be 3 times as long as the other. How long will each piece be?

10. A back yard 32 ft. wide and 48 ft. long is to have a tight board fence built along the two sides and across one end. The boards are set upright and are 9 in. wide and cost 8¢ apiece. What is the cost of the boards for the entire fence?

11. Two boys agree to wash 15 windows for 90¢. One of the boys washes 9 windows and the other 6. How much should each boy be paid?

12. A girl is paid 15¢ an hour for her work. On Monday she began at 8 A.M. and quit at 4 P.M., on Tuesday she worked from 9 A.M. till 6 P.M., and on Wednesday from 7 A.M. till 5 P.M., taking an hour off each day for lunch time. How much was due her for the three days' work?

13. A boy begins work on Monday morning, March 27th, and works every day but Sundays until the evening of the 15th of April. How many days does he work?

14. A certain room is 15 ft. long and 13 ft. wide. Picture molding which runs around the room costs 5¢ a foot. How much does the molding for the room cost?

15. A boy counts 65 telegraph poles while the train he is on runs for four minutes. If the poles are 165 feet apart, how many miles an hour is the train running?

16. At a picnic 12 gallons of ice cream, cut 6 pieces to the quart, were ordered and eaten by 211 children. How many had two pieces?

17. A boy is required to write a composition of about 500 words. He finds that he writes about 8 words to the line and 20 lines to the page. How many pages must he write?

Drill Tables

2. The addition table. To solve problems skillfully the pupil must be able to compute accurately and rapidly. Speed and accuracy are impossible until the pupil has developed speed in giving the following tables.

These exercises are to be practiced by the pupils and used by the teacher for rapid drill for a few minutes at the beginning of each recitation until the suggested speed and accuracy have been secured.

Practice giving these sums orally until they can be given in less than 30 seconds.

3	5	8	7	8	2	5	6	1	3	9	
2	5	6	7	9	7	4	7	1	5	4	
1	9	2	7	6	1	5	9	4	1	8	
2	3	4	5	6	3	8	2	6	8	8	
6	1	4	1	5	8	9	9	3	7	9	
3	9	4	4	2	7	9	5	3	4	6	
2	3	1	2	6	1	9	4	8	1	8	2
2	7	7	6	5	5	7	3	4	6	3	8

3. The subtraction table. Practice giving these differences until they can be given in less than 40 seconds.

6	12	9	2	13	14	11	11	7	8	7
3	7	4	2	9	8	9	8	2	4	5
13	12	15	16	9	1	17	14	11	14	12
4	9	7	9	1	1	8	6	2	5	4
9	10	13	12	3	11	12	15	8	8	17
3	1	5	6	2	4	3	6	8	5	9
16	12	5	6	4	13	16	3	11	14	5
7	5	2	6	4	6	8	1	3	7	5

Practice giving these differences until they can be given in less than 40 seconds.

7	10	8	11	15	9	10	8	15	8	7
6	3	1	6	8	7	2	6	9	3	1
9	11	18	9	12	8	9	6	10	7	10
6	5	9	2	8	7	5	1	9	3	4
6	10	9	5	6	13	3	13	5	6	11
2	7	9	1	5	8	3	7	4	4	7
4	9	10	14	5	2	7	10	7	8	10
3	8	5	9	3	1	4	6	7	2	8

4. The multiplication table. Practice giving these products until each group can be given in less than 35 seconds. Also practice writing the products until each group can be written in less than 60 seconds.

1

2	9	7	3	2	9	9	11	8	5	1	12	4
2	6	4	3	8	5	9	7	7	2	4	4	4
1	8	1	8	4	9	1	6	2	1	3	6	3
9	3	6	4	3	7	5	5	6	7	7	12	11
6	1	8	12	1	4	11	9	9	12	5	7	1
3	10	8	7	8	6	6	10	2	8	8	10	3

2

4	6	11	10	7	5	2	12	9	8	1	9	3
10	6	5	8	5	12	4	9	3	11	2	4	5
2	1	12	6	11	5	3	2	9	8	10	12	7
10	1	3	7	1	4	10	7	11	9	10	1	7
6	8	12	12	5	10	2	11	3	5	11	11	12
10	6	11	10	10	11	12	4	2	5	2	11	12

5. The division table. Practice giving these quotients till they can be given in less than one minute. Also practice writing them until they can be written in less than two minutes.

4)32	9)36	11)132	11)66	8)48	8)88
9)27	7)49	12)84	9)108	12)48	8)64
10)100	5)60	8)72	7)35	9)99	11)121
8)56	11)77	11)55	11)22	9)81	3)21
6)36	9)45	11)99	6)42	11)44	6)30
12)36	7)28	5)40	9)54	7)63	12)96
9)18	12)120	8)32	4)36	6)66	6)48
8)24	11)88	6)24	3)27	7)84	12)108
4)48	12)60	9)72	5)35	7)56	11)33
8)80	7)77	12)24	5)55	7)21	4)20
5)45	7)42	4)44	5)30	3)36	4)28
8)40	6)54	9)63	8)96	3)18	7)14

Exercise 2. Problems for solution

1. For a physical training drill the pupils of a school are arranged in 9 rows with 28 pupils in a row. How many rows could be made with 36 pupils in a row?

2. In a school of 427 pupils the physical director arranges as many as possible of them in 24 equal rows. How many pupils are left out? How many more pupils would be needed to keep the 24 rows equal if all the 427 pupils were used?

3. Each of six boys puts all the money in his bank into a common fund to buy pigeons. The first has 32¢, the second 28¢, the third 49¢, the fourth 45¢, the fifth 38¢, the sixth 35¢. How many pairs of pigeons can they buy at 45¢ a *pair?*

4. The top of a tree is 38 ft. from the ground and its deepest root extends 8 ft. underground. How far is it from the end of its deepest root to the top of the tree? Its first branch is 13 ft. from the ground. How far is it from the lowest branch to the top?

5. Mr. Brown bought his winter's supply of coal at $3.65 a ton. It came in seven loads weighing 5317 lb., 4142 lb., 5328 lb., 4618 lb., 5069 lb., 4354 lb., and 3728 lb. How much was his coal bill?

6. The usual size of a tennis court is 78 ft. by 36 ft. A boy agrees to strip the sod from a court for 3¢ for each 2 sq. yd. and to roll the ground thoroughly for 1¢ for each 6 sq. yd. How much does he receive for the job?

7. It takes a boy 54 minutes to ride 6 miles on his bicycle. At that speed how long does it take him to go a mile? How many yards does he go in a minute?

8. A man can row a boat upstream 2 miles in an hour, and he can row downstream 5 miles in an hour. How long will it take him to row 10 miles upstream and return? How long to row 10 miles downstream and return?

9. A train travels from A to B, a distance of 171 miles, in 6 hours. Running at the same speed, how long will it take it to return $\frac{1}{3}$ of the distance?

10. Soldiers marching take steps about 2 ft. 6 in. long. If they take 120 steps a minute, how long will it take them to go 4 miles?

11. A man earns 35¢ an hour. He works 26 days in a month, 8 hours each day. He rides to and from work, buying 6 tickets for a quarter. He pays a quarter each day for his lunch. How much does he have each month for other expenses?

12. *Practice the multiplication table on page 6 until the products can be given in the time suggested.*

6. Exercises for practice in addition. Find the sums of these short columns on five successive days. Record the time required and note your improvement. Do not copy the examples. Place a paper below them and write the sums upon it. Add consecutive numbers when their sum is 10. Thus, in the first column think 7, 17, 26, 31.

1.	2.	3.	4.	5.	6.	7.	8.	9.	10.	11.	12.	13.	14.	15.
5	9	8	6	2	4	8	7	9	3	6	9	4	6	9
9	3	5	9	9	7	7	9	4	7	4	5	8	7	6
4	6	7	2	5	6	8	4	3	5	6	8	2	4	5
6	5	6	7	8	4	2	5	8	8	9	2	7	6	5
7	7	9	4	4	9	6	8	3	7	5	7	4	5	8

Practice for speed and accuracy in finding the following sums. To check your results add once beginning at the bottom of the column and once beginning at the top.

1.	2.	3.	4.	5.	6.	7.	8.
486	392	738	984	171	196	917	410
213	305	310	592	235	800	705	396
357	814	986	301	968	347	256	967
374	977	896	135	401	967	835	284
867	263	245	409	968	225	478	753
348	596	357	754	513	319	998	214

Write the sums of the following numbers as rapidly as possible. In the first exercise get the sum by adding 42 and 30 and 5. Think 42, 72, 77.

1.	2.	3.	4.	5.	6.	7.	8.	9.	10.	11.	12.
42	78	19	43	85	91	54	38	27	92	65	74
35	15	68	72	24	18	69	81	59	94	29	46

13.	14.	15.	16.	17.	18.	19.	20.	21.	22.	23.	24.
18	23	41	36	63	57	46	17	33	63	76	34
82	59	87	96	78	28	85	86	92	98	58	89

7. Cash accounts. It is important for every one who receives and spends money to be able to keep a cash account. In a cash account are recorded the amount and the date of each sum received and of each sum paid out. The difference between the sum of the receipts and the sum of the payments is called the **balance.**

EXAMPLE. Write and balance the following account: Receipts: July 1, cash on hand, \$2.85; July 6, picking berries, 45¢; July 15, errands, 25¢; July 20, mowing lawn, 50¢; July 30, birthday present, \$1. Payments: July 2, fishing line, 25¢; July 4, expenses for picnic, 40¢; July 18, cap, 75¢. Find the balance July 31.

		Receipts					*Payments*		
1921					1921				
July	1	*Cash on hand*	2	85	*July*	2	*Fishing line*		25
	6	*Picking berries*		45		4	*Expenses for picnic*		40
	15	*Errands*		25		18	*Cap*		75
	20	*Mowing lawn*		50		31	*Balance*	3	65
	30	*Birthday present*	1	00					
			5	05				5	05
Aug.	1	*Cash on hand*	3	65					

Exercise 3

Write and balance each of the following accounts, ruling the paper as in the example above:

1. Receipts: Jan. 1, 1923, cash on hand, \$3.56; Jan. 1, gift, 50¢; Jan. 10, shoveling snow, 25¢; Jan. 17, errands, 20¢; Jan. 26, shoveling snow, 25¢; Jan. 31, selling papers this month, \$3. Payments: Jan. 2, sharpening skates, 25¢; Jan. 8, moving pictures, 20¢; Jan. 12, mittens, 50¢; Jan. 18, car fare, 15¢; Jan. 25, relief fund, 25¢. Balance, Jan. 31.

2. Receipts : May 1, cash on hand, \$1.75 ; May 4, caring for children, 25¢ ; May 6, washing dishes, 30¢ ; May 12, delivering milk, \$1.15 ; May 20, picking strawberries, 40¢ ; May 28, errands, 15¢. Payments : May 6, ribbon, 40¢ ; May 9, car fare, 10¢ ; May 12, ice cream soda, 20¢ ; May 18, pencil, 10¢ ; May 30, picnic expenses, 50¢. Balance, May 31.

3. Receipts : Aug. 1, cash on hand, \$57 ; Aug. 7, week's salary, \$20 ; Aug. 14, week's salary, \$20 ; Aug. 21, week's salary, \$20 ; Aug. 21, overtime, \$3.50 ; Aug. 28, week's salary, \$20. Payments : Aug. 3, shirts, \$3.50 ; Aug. 9, trip, \$4.80 ; Aug. 31, room rent, \$10 ; Aug. 31, board, \$20. Balance, Aug. 31.

4. Receipts : Sept. 1, cash on hand, \$937.40 ; Sept. 5, J. H. Sims, \$89.60 ; Sept. 12, R. S. Myers, \$32.48 ; Sept. 26, J. Minnick, \$148.29 ; Sept. 30, C. B. Jones, \$200. Payments : Sept. 9, J. J. Fitch, \$38.65 ; Sept. 15, Felix Post, \$9.83 ; Sept. 20, K. G. O'Neil, \$60. Balance, Sept. 30.

5. For part of one year a boy kept an account of the receipts and expenditures for his cow as follows :

Expenditures : Sept. 9, 100 lb. dairy feed at \$1.65 ; Oct. 1, 100 lb. dairy feed at \$1.75 ; Oct. 20, 100 lb. cracked corn at \$2 ; Oct. 20, 100 lb. bran at \$1.65 ; Oct. 20, 50 lb. oil meal at \$2.30 a hundred ; Nov. 7, 25 lb. salt, \$.25 ; Nov. 7, pasture, \$5.15, hay, \$4.80 ; Nov. 18, 100 lb. oil meal, \$2.50, 200 lb. cracked corn, \$4.50, 200 lb. bran, \$3.40, 1025 lb. hay at \$.75 a hundred, 2 bu. oats, \$1 ; Dec. 3, clover, 4 bales, \$3.

Receipts : Nov. 1, milk sold to date, 780 lb. at \$4.35 a hundred ; Nov. 29, 552 lb. milk at \$4.35 ; Dec. 6, 121 lb. milk at \$4.35 ; Dec. 13, 125.3 lb. at \$4.35 ; Dec. 20, 117 lb. at \$4.35; Dec. 31, 125 lb. at \$4.35; Jan. 5, 103 lb. at \$4.35; Jan. 12, 118 lb. at \$4.35; Jan. 19, 115 lb. at \$4.35.

Balance the account Jan. 20.

8. Finding averages. If Tom earned 30¢ on Monday, 45¢ on Tuesday, and 33¢ on Wednesday, how much did he earn in the three days?

If he had earned equal amounts on each of the three days, how much must he have earned each day to have earned the $1.08?

This 36¢ is called the **average** amount that he earned for the three days.

A girl saved 15¢ one week, 35¢ the second week, 28¢ the third week, and 13¢ the fourth week. Find the average amount per week saved for the four weeks.

The average of several numbers is found by dividing their sum by the number of them.

Exercise 4

1. A cow gives 22 lb. of milk on Monday, 18 on Tuesday, $19\frac{1}{2}$ on Wednesday, 21 on Thursday, 23 on Friday, $21\frac{1}{2}$ on Saturday, and 19 on Sunday. Find her average daily yield.

2. Ask the age of each pupil in your room. Find the average age of the pupils in your room.

3. Ask the teacher to tell you the number of pupils present in your room each day for the past month. Find the average daily attendance in your room for the past month.

4. Mrs. Brown's hens have the following daily record of eggs produced : 54, 32, 60, 41, 56, 28, 63, 38, 50, 39, 64, 29, 55, 34, 58, 42. Find the average daily number of eggs.

5. The average daily number of eggs for Mrs. Brown's hens for the next 20 days was $42\frac{3}{4}$ eggs. How many eggs did they lay in the 20 days?

6. A train ran 30 miles the first hour, 35 miles the second hour, 24 miles the third hour, 28 miles the fourth hour, and 31 miles the fifth hour. What was its average rate per hour *for the five hours?*

7. The annual rainfall for a certain place for the years 1904 to 1913 inclusive was 26.14 in., 35.36 in., 30.37 in., 35.10 in., 34.83 in., 43.22 in., 26.86 in., 33.83 in., 29.67 in., and 27.11 in. How much below the average was the rainfall of the year having the least rainfall? How much above the average was that of the year having the greatest rainfall?

8. In his examinations Arthur has made the following grades : arithmetic 80, geography 70, grammar 77, and spelling 68. He is still to be examined in history. What grade must he make in history in order to have an average of 75?

9. Practice giving the sums in the first 15 exercises of page 9.

Exercise 5. Problems of the farmer

1. Mr. Henry purchased $13\frac{1}{2}$ tons of fertilizer of Mr. Shubert at $18.50 a ton. He sold Mr. Shubert $246\frac{1}{2}$ bushels of potatoes at 45¢ a bushel and paid the balance in cash. How much cash did he pay?

2. Mr. Black bought 160 acres of land at $120 an acre. His taxes for the year are $1 an acre and the repairs cost him $1 an acre. He can rent the farm to a tenant for $8 an acre cash rent, but chooses to rent it for crop rent. He gets one-half of the corn, one-third of the broom corn, one-third of the oats, one-third of the wheat, one-half of the hay, and $5 an acre for the pasture. The tenant raises 40 bushels of corn per acre on 40 acres ; one-fourth of a ton of broom corn per acre on 20 acres ; 20 bushels of wheat per acre on 30 acres ; 40 bushels of oats per acre on 25 acres ; 2 tons of hay per acre on 25 acres. He uses 18 acres for pasture. The corn is sold at 60¢ a bushel, the broom corn at $150 a ton, the oats at 42¢ a bushel, the wheat at $1.10 a bushel, and the hay at $12 a ton. How much more does Mr. Black receive than if he had rented the farm for cash rent?

3. Six one-acre plots of ground produced the following amounts of oats in five consecutive years:

PLOT NO.	1909	1910	1911	1912	1913
1	56	67	48	63	68
2	38	45	36	48	52
3	24	32	21	35	41
4	44	52	48	58	60
5	36	41	34	44	48
6	40	48	45	52	55

Find the average number of bushels per acre for the six plots the first year; for each of the other years.

Find the average annual yield of the first plot for the five years; also for each of the other plots.

4. A herd of 18 cows has the following milk record in pounds for a year:

8372	10,397	6482	7538	5682	9430
10,843	12,963	7645	9602	7268	12,458
9873	8629	11,062	6483	9463	7526

Find the average annual production of these cows.

5. Mr. Hopkins raised 1744 bushels of corn on 42 acres. What was the average yield per acre?

6. It is estimated that more than five billion tons of freight are moved over the public roads of the United States each year. It is also estimated that the average distance each ton is hauled is about ten miles and that it costs about 23¢ to haul one ton one mile. What is the total cost of hauling this freight? It is estimated that the cost of hauling one ton of freight one mile could be reduced to 8¢ if we had good roads. How much would be saved by good roads *each year?* This amount saved would build how many *miles of good roads* at $10,000 a mile?

Exercise 6. School garden problems

1. Each of the 40 pupils in the seventh grade of a certain school planted a garden 30 ft. long and 10 ft. wide according to the plat given in Figure 1. Rows numbered 1 to 14 inclusive were one foot apart. The remaining rows were 2 ft. apart. They estimated the yield of each garden to be as given below. (A bunch contains ½ doz. vegetables.)

	AMOUNT	PRICE
Parsnips .	1 every 3 in.	5¢ a bunch
Chard . .	2 lb. per foot	5¢ a pound
Lettuce .	1 lb. per foot	8¢ a pound
Radishes .	1 doz. per foot	5¢ a bunch
Peas . .	8 gal. per 100 ft.	30¢ a gallon
Onions . .	1 every 2 in.	5¢ a bunch
Beets . .	1 every 3 in.	5¢ a bunch
Carrots .	1 every 3 in.	5¢ a bunch
Head lettuce . .	20 heads, 1½ lb. each	10¢ a pound
Beans . .	2 bu. per 100 ft.	$1 a bushel
Cabbage .	12 heads, 4 lb. each	6¢ a pound
Tomatoes .	80 lb.	5¢ a pound
Cucumbers	½ bu.	$1 a bushel

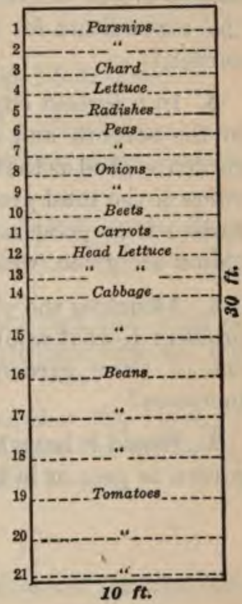

FIG. 1.

Find the estimated value of the total yield of one garden.

2. The pupils harvested and sold all the produce from their gardens. The best ten gardens produced as follows: $23.40, $21.06, $19.75, $18.65, $18.10, $17.40, $17.15, $16.55, $15.40, $15.10. Four pupils failed to care for their gardens and they produced nothing. The poorest ten producing gardens produced as follows: $1.15, $2.35, $4.45, $6.10, $6.60, $7.25, $8.40, $9.65, $11.10, $13.40. Find the average value of the produce of the best ten gardens; of the poorest ten producing gardens; of the poorest ten gardens.

3. Find the average value of the produce of the 20 gardens whose returns are given above.

4. If the family of the pupil growing the best garden consumed $70.20 worth of vegetables in a year, what part of this expenditure for vegetables did the pupil produce in his garden?

5. In a certain city of 20,000 people there are 720 pupils in the seventh and eighth grades. If each pupil raises a garden as good as the best mentioned in problem 2, what is the value of the total produce of these gardens? If each pupil's garden is as good as the average of the 20 gardens whose values are given, what is the total value of their produce?

6. Assuming the yields to be the same as the estimates in problem 1, what would have been the value of the crop from one of these gardens if planted entirely in onions? In tomatoes?

7. Would it have been more profitable to plant the whole garden in peas or in beans? How much?

CHAPTER II

THE FORMULA. REVIEW OF FRACTIONS

9. Some useful terms. The result of adding two or more numbers is called their **sum**. The numbers added are called **addends**.

When one number, the **subtrahend**, is subtracted from another number, the **minuend**, the result is called their **difference** or **remainder**.

When two or more numbers are multiplied the result is called their **product**. The numbers multiplied together are called the **factors** of the product.

When one number is divided by another the result is called their **quotient**. The number divided is the **dividend**. The number by which it is divided is the **divisor**.

Exercise 7

1. The sum of two numbers is 15 and one of the numbers is 6. What is the other number?

2. Two addends together make 87. One of them is 29. What is the other?

3. The sum of three numbers is 37. Two of the numbers are 12 and 14. What is the third?

4. If the sum of two numbers and one of the numbers are known, how can the other be found?

5. If you know the sum of three numbers and two of the numbers, how can you find the other?

6. The difference of two numbers is 5 and the smaller number is 13. Find the larger number.

7. The difference of two numbers is 7. One of the numbers is 19. The other may be either of two numbers. Find both of them.

8. If you know the difference of two numbers and the smaller one, how can you find the larger one?

9. If you know the larger of two numbers and their difference, how can you find the smaller one?

10. The product of two numbers is 63, and one of the numbers is 7. Find the other factor.

11. The product of two factors is 714 and one of the factors is 17. What is the other factor?

12. The product of three factors is 5304 and two of them are 13 and 17. Find the third factor.

13. When a product and one of its two factors are known, how can the other factor be found?

14. When a product of three factors and two of the factors are known, how can the third factor be found?

15. The quotient of two numbers is 6. The dividend is 42. Find the divisor.

16. The quotient of two numbers is 18. The divisor is 35. Find the dividend.

17. When told the quotient of two numbers and the divisor, how can the dividend be found?

18. When told the quotient of two numbers and the dividend, how can the divisor be found?

10. Problems without numbers. In the "Directions for solving problems" you were told to think the process by which the things asked for may be found. This is often difficult but it can be done even if there are no numbers in the problem with which to compute the answer.

Tell what process you would use to find how much a boy would earn in a week if you knew how much he earns a day.

Exercise 8

1. A man bought 7 head of cattle at $32.50 a head and sold them at $54 a head. How much did he make on them?

2. A certain number of articles are bought at a certain price each and sold at a certain higher price each. In what two ways can you find the total profit? Which is the easier?

3. If you are told how much a boy earns each week and how much he spends each week, how can you find out how much he saves in a year?

4. If you are told how much a man earns in a year, what else do you need to know in order to find out how much he earns each day he works? If you are told this, how do you then find out how much he earns a day? If told how many hours a day he works, how can you then find out how much he earns an hour? In a minute?

5. If told the number of oranges in a box and the selling price a dozen, how can you find the selling price of the box of oranges?

6. What do you need to know in order to find out how many steps you take in walking a mile?

7. How can you find out how many steps you take in walking a mile if you know how many feet you go at each step?

8. What measurement would you make to find how many revolutions a wheel makes in going a mile?

9. The circumference of a certain wheel is 12 feet. How many revolutions does it make in going a hundred yards?

10. If told that two-thirds of the pupils in your room are girls, what else would you need to know in order to compute the number of boys in the room?

11. If you know the cost of several tons of coal and the cost of one ton, ho [illegible] the number of tons bought?

12. If told the width of a piece of cloth, how can you find the number of strips, each 4 inches wide and as long as the piece, which can be cut from it?

13. How can you find out how many cords in a pile of four-foot wood?

14. What measurements would you make to find how many books all of the same size a certain shelf would hold?

15. How can you find the average of 4 numbers?

11. The cost formula.

1. If a yard of cloth costs $8\frac{1}{3}$¢, how much do 9 yards cost?

2. If the price of hay is \$12.50 a ton, how much do $2\frac{1}{2}$ tons cost?

3. If the price of an article is known, how is the cost of a certain number of these articles found?

4. If p represents the price of an article and n is the number bought, what is the cost of the n articles?

The cost of a number of articles is equal to the number bought times the price of each.

If the cost is represented by C then this principle may be condensed into the formula:

$$C = n \times p.$$

This is a short form for the statement, cost equals the number times the price of one.

Such a short form is called a **formula.**

Formulas are especially useful in solving a large number of problems of the same kind.

EXAMPLE. Find the cost, C, if $n=3$ and $p=5$¢.

SOLUTION. $C=n\times p=3\times 5¢=15¢$.

When numbers are represented by letters their product is usually expressed by writing them together with no sign between them. Thus, np means n times p; $5x$ means 5 times x.

Exercise 9

Use the formula $C = np$ to find the values of C for the given values of n and p.

	n	p	C
1.	10	\$2	\$20
2.	18	35¢	
3.	$2\frac{1}{2}$	10¢	
4.	5	\$18	
5.	24	$3\frac{1}{2}$¢	
6.	75	\$2	
7.	40	$4\frac{1}{5}$¢	
8.	48	$3\frac{1}{3}$¢	

9. A woman buys three cantaloupes for a quarter. What is the average price?

10. Fifteen tons of coal cost \$48. What is the price per ton?

11. If told the cost of a certain number of things, how may the price of one be found? Make a formula of the rule you have just stated.

This formula may be written in either of two ways:

$p = C \div n$, read "p equals C divided by n";

or $p = \frac{C}{n}$, read "p equals C over n," or "p equals C divided by n."

Complete this table, giving values of p for the given values of C and n.

	n	p	C
12.	5		\$235
13.	$\frac{1}{3}$		9¢
14.	$\frac{2}{5}$		\$170
15.	10.5		\$37.50
16.	247		\$16,475

17. Read and write the product of x and y; of 7 and c; of m, n, and k; of r, s, and 3; of t, 9, and d.

12. Making formulas. Since it is generally much easier to remember a formula than the much longer principle or rule, most of the rules of arithmetic are stated as formulas.

The rule for finding the cost of a number of articles when the price of one and the number of articles are known has been stated in the formula $C=np$.

To state a rule as a formula it is necessary to represent the numbers mentioned in the rule by letters, and to indicate the processes to be performed by the proper signs.

The sum of a and b is written $a+b$. The difference when b is subtracted from a is written $a-b$. The product of a and b is written either $a \times b$ or ab, generally the latter. The quotient of a by b is written either $a \div b$ or $\frac{a}{b}$.

EXAMPLE. One-fourth of the desks were taken from a school building having five rooms. The first room contained n desks, the second 15 desks, and each of the other three rooms contained d desks. How many desks were taken away?

SOLUTION. In the last three rooms there were $3d$ desks.

In all the five rooms there were $3d+n+15$ desks.

The number taken away $= \frac{1}{4}$ of $(3d+n+15) = \frac{3d+n+15}{4}$.

Exercise 10

1. Write the sum of c and d; of m, n, and r; of 2 and m; of 3, a, and b; of 7, b, c, and x. Read each of these sums.

2. In one field there are 12 acres and in another n acres. How many acres in both fields?

3. There are n children in one grade. Seven of them are boys. How many girls are there?

4. Write the number which is twice k.

5. How many inches in 6 ft.? In f ft.?

6. *How many* ears have h horses? How many feet?

7. How many feet have m men? How many feet have h horses and m men?

8. How many quarts in n gallons? How many gallons in n quarts?

9. How many feet in y yd. 1 ft.? How many inches?

10. How many oranges can be bought for 80¢ at r cents apiece? For r cents at 5 cents apiece?

11. If the distance around a square is p, how long is one side?

In making formulas, first think the method by which the number required for the answer may be found; then, using a letter to represent the number required, indicate the processes by which it is found.

EXAMPLE 1. Make a formula for finding the number of inches in f ft. n in.

SOLUTION. Let i represent the number of inches in f ft. n in. To find this number of inches we must multiply the number of feet by 12 and add the number of inches.

Then $$i = 12f + n.$$

EXAMPLE 2. Make a formula for finding the total weight of a basket containing 3 buckets of lard and 5 cans of corn, if the weight of the empty basket, the weight of one can of lard, and the weight of one can of corn are known.

SOLUTION. To find the total weight we must multiply the weight of one bucket of lard by 3, the weight of one can of corn by 5, and add these two weights to the weight of the basket. Represent the weight of the empty basket by b, the weight of one bucket of lard by l, the weight of one can of corn by c, and the total weight by W. Then the weight of the 5 cans of corn is $5c$, the weight of the 3 buckets of lard is $3l$, and the weight of the empty basket is b. The total weight, W, is therefore the sum of b, $3l$, and $5c$.

The formula is, therefore, $W = b + 3l + 5c$.

Exercise 11

1. Write a formula for finding the sum, s, of two numbers a and b.

2. Write a formula for finding the weight, w, of a barrel of crackers, if the weight of the barrel is b and the weight of the crackers is c.

3. Make a formula for finding the number of gallons of water, g, that run through a pipe in 25 minutes at the rate of n gallons a minute.

4. Make a formula for finding the number of cents, C, in d dollars and 4 dimes ; for finding the number of cents, C, in d dollars, n dimes, and 3 cents.

5. Find the average of the two numbers 7 and 9. How is the average of any two numbers found?

6. Make a formula for finding the average, A, of any two numbers x and n.

7. Make a formula for finding the combined weight of a hen and 9 chickens, if the weight of the hen is h pounds and the weight of each chicken is c pounds. Represent the combined weight by W.

8. A boy walks 3 mi. each hour for 2 hours. How far does he walk in the two hours?

9. How far does a train run in 7 hours if it runs 25 miles each hour?

10. A boy runs 8 yd. each second for $12\frac{1}{2}$ seconds. What distance does he run in that time?

The distance an object moves in one unit of time is called its rate of motion.

11. A river flows at the rate of $2\frac{1}{2}$ mi. an hour. How far will an object floating in it move in 8 hr. 20 min.?

12. How long will it take a man to drive 20 mi. at the rate of 8 mi. an hour?

13. How long will it take a train to run 640 mi. at the rate of 32 mi. an hour?

14. How can you find the distance an object moves in a given time at a given rate?

15. Make a formula for finding the distance, d, which an object moves at the rate, r, in the time, t.

16. Find the rate of a bicyclist who rides 32 mi. in 4 hr.

17. Find the rate of a train which runs 414 mi. in 9 hr.; of a train which runs m mi. in 9 hr.; of a train which runs m mi. in h hr.

18. Calling the rate r, the whole distance d, and the time t, state the formula for finding the rate when the time and the distance are known.

19. Calling the weight of a basket b, the weight of an egg e, and the weight of an apple a, make a formula for finding the total weight, W, of a basket containing a dozen eggs and 25 apples.

It is sometimes convenient to use A and a in the same formula to stand for different numbers. A is then read "A major" and a is read "a minor."

20. Make a formula for finding one factor, F, of a product, when the product, p, and the other factor, f, are known.

21. Make a formula for finding the dividend, D, when the divisor, d, and the quotient, q, are given.

22. Make a formula for finding the dividend, D, when the divisor, d, the quotient, q, and the remainder, r, are given.

23. How can the divisor be found when the dividend, quotient, and remainder are known? State this rule as a formula.

24. State the formula for finding the area, A, of a rectangle whose length is l and width w.

13. Formulas for practice. These formulas which you have had should be remembered.

1. $C=np$.
2. $A=lw$.
3. $d=rt$.
4. $V=lwh$.

Formula 1 is the cost formula. Formula 2 is the rule for the area of a rectangle whose length and width are given. Formula 3 is the formula for the distance passed over by a body moving at the rate r for the time t. Formula 4 is the rule for the volume of a rectangular solid.

Other formulas which you have made but which need not be remembered are:

(*a*) $D=dq$. Formula to find the dividend when divisor and quotient are given.

(*b*) $D=dq+r$. To find the dividend when there is a remainder.

(*c*) $T=12e+25a+b$. To find the total weight, T, of a basket with 12 eggs and 25 apples.

(*d*) $F=p\div f$. To find one factor of a given product when the other factor is given.

(*e*) $A=\frac{x+n}{2}$. To find the average of two given numbers.

If given the values of the letters supposed to be known in these formulas, these values may be substituted for the letters and the value of the letter on the left may be found by performing the indicated operations.

EXAMPLE. Using formula *c*, find the total weight of a basket containing a dozen eggs weighing 2 ounces each and 25 apples weighing 3 ounces each, the basket weighing 14 ounces.

SOLUTION. We know that $e=2$, $a=3$, and $b=14$.

Substituting 2 for e, 3 for a, and 14 for b,

$$W=12\times2+25\times3+14=24+75+14=113.$$

113 ounces = 7 lb. 1 oz.

Exercise 12

1. Find D of formula (a), if $d=15$ and $q=6$.
2. Find C of formula 1, if $n=20$ and $p=37\frac{1}{2}$.
3. Find F of formula (d), if $p=108$ and $f=6$.
4. Find T of formula (c), if $e=1.9$, $a=2.1$, and $b=10$.
5. Find D of formula (b), if $d=14$, $q=8$, and $r=9$.
6. Find A of formula 2, if $l=12\frac{1}{2}$ and $w=3\frac{1}{5}$.
7. Find d of formula 3, if $r=\frac{1}{5}$ and $t=6$.
8. Find T of formula (c), if $e=2.1$, $a=2$, and $b=8$.
9. Find D of formula (b), if $d=247$, $q=43$, and $r=203$.
10. Find A of formula (e), if $x=974$ and $n=863$.
11. Find T of formula (c), if $e=2$, $a=3.6$, and $b=12.8$.
12. Find F of formula (d), if $p=60\frac{2}{3}$ and $f=7$.
13. Find V of formula 4, if $l=5.8$, $w=.3$, and $h=25$.
14. Find d of formula 3, if $r=12.7$ and $t=65$.

Use the formulas to find the results in the following :

15. If a certain number is divided by 67, the quotient is 19 and the remainder 46. Find the number.

16. Find the volume of a piece of steel 6 in. long, .3 in. wide, and .2 in. thick.

17. How far does sound travel in 1 minute if it travels 1080 ft. in one second?

18. The product of two numbers is 9.864 and one of the numbers is 12. Find the other number.

19. The product of two numbers is $\frac{15}{32}$ and one of the numbers is 105. Find the other number.

20. Find the average of $2\frac{1}{6}$ and $3\frac{1}{4}$.

21. Find the average of $17\frac{4}{5}$ and $37\frac{2}{3}$.

22. What is the average of two numbers a and b?

Review of Fractions

14. Reduction of fractions.

1. What part of 6 is 3?
2. 7 is what part of 42?
3. 10 is what part of 15?
4. What part of 5 is 4?
5. How can you find what part one number is of another?

Example. 18 is what part of 54?

Solution. 18 is $\frac{18}{54}$ of 54.

$\frac{18}{54}$ may be reduced to its lowest terms by dividing both numerator and denominator by 18.

Therefore 18 is $\frac{1}{3}$ of 54.

A fraction is reduced to its lowest terms by dividing both numerator and denominator by their greatest common factor.

Exercise 13

Practice until you can read these twenty-five fractions in their lowest terms in 90 seconds.

1. $\frac{4}{6}$.	7. $\frac{10}{15}$.	13. $\frac{34}{51}$.	19. $\frac{20}{30}$.
2. $\frac{25}{35}$.	8. $\frac{12}{18}$.	14. $\frac{15}{20}$.	20. $\frac{15}{18}$.
3. $\frac{14}{21}$.	9. $\frac{6}{9}$.	15. $\frac{16}{20}$.	21. $\frac{30}{36}$.
4. $\frac{6}{12}$.	10. $\frac{12}{15}$.	16. $\frac{18}{22}$.	22. $\frac{35}{45}$.
5. $\frac{18}{24}$.	11. $\frac{20}{25}$.	17. $\frac{20}{28}$.	23. $\frac{21}{56}$.
6. $\frac{9}{15}$.	12. $\frac{36}{42}$.	18. $\frac{6}{21}$.	24. $\frac{9}{36}$.

25. In a certain schoolroom there are 12 boys and 18 girls. What part of the pupils in the room are boys? What part are girls?

26. A basketball player tries 27 throws at the basket and misses 12 times. What part of his throws are successful?

27. A ball team wins 14 games and loses 6 the first month. The second month it wins 15 games and loses 9. What *part of the games* played did it win the first month? The *second month*?

15. Exercises for practice. The following exercise is not meant to be assigned as a lesson, but to be used for a few minutes of vigorous drill each day until the suggested standard of skill is reached.

Exercise 14

Read the following equations, supplying the missing numerators, in $2\frac{1}{2}$ minutes.

1. $\frac{3}{4}=\frac{?}{12}$.
2. $\frac{2}{5}=\frac{?}{25}$.
3. $\frac{6}{7}=\frac{?}{21}$.
4. $\frac{1}{4}=\frac{?}{100}$.
5. $\frac{4}{5}=\frac{?}{100}$.
6. $\frac{5}{8}=\frac{?}{100}$.
7. $\frac{3}{8}=\frac{?}{48}$.
8. $\frac{3}{10}=\frac{?}{40}$.
9. $\frac{2}{3}=\frac{?}{18}$.
10. $\frac{1}{3}=\frac{?}{18}$
11. $\frac{2}{3}=\frac{?}{27}$.
12. $\frac{1}{2}=\frac{?}{100}$.
13. $\frac{1}{3}=\frac{?}{100}$.
14. $\frac{5}{7}=\frac{?}{14}$.
15. $\frac{5}{12}=\frac{?}{60}$.
16. $\frac{2}{7}=\frac{?}{63}$.
17. $\frac{5}{6}=\frac{?}{24}$.
18. $\frac{4}{7}=\frac{?}{42}$.
19. $\frac{5}{6}=\frac{?}{60}$.
20. $\frac{1}{5}=\frac{?}{100}$.
21. $\frac{2}{3}=\frac{?}{100}$.
22. $\frac{3}{5}=\frac{?}{35}$.
23. $\frac{7}{12}=\frac{?}{96}$.
24. $\frac{7}{8}=\frac{?}{40}$.
25. $\frac{9}{10}=\frac{?}{50}$.
26. $\frac{5}{8}=\frac{?}{56}$.
27. $\frac{3}{4}=\frac{?}{100}$.
28. $\frac{1}{8}=\frac{?}{100}$.
29. $\frac{4}{9}=\frac{?}{45}$.
30. $\frac{5}{6}=\frac{?}{72}$.

Write the following sums and differences each day for four days. By how much have you reduced the time it takes you?

31. $\frac{1}{2}+\frac{1}{4}$.
32. $\frac{1}{2}-\frac{1}{3}$.
33. $\frac{1}{3}+\frac{1}{6}$.
34. $\frac{5}{12}+\frac{2}{3}$.
35. $\frac{1}{3}+\frac{1}{5}$.
36. $\frac{5}{6}+\frac{4}{9}$.
37. $\frac{4}{5}-\frac{5}{12}$.
38. $\frac{1}{3}+\frac{1}{2}$.
39. $\frac{3}{4}+\frac{1}{2}$.
40. $\frac{3}{4}+\frac{1}{8}$.
41. $\frac{9}{10}-\frac{1}{5}$.
42. $\frac{5}{6}+\frac{3}{8}$.
43. $\frac{4}{5}-\frac{2}{3}$.
44. $\frac{4}{5}-\frac{1}{2}$.
45. $\frac{2}{3}-\frac{1}{2}$.
46. $\frac{1}{2}+\frac{1}{8}$.
47. $\frac{5}{6}-\frac{3}{4}$.
48. $\frac{7}{12}-\frac{1}{4}$.
49. $\frac{2}{3}+\frac{3}{5}$.
50. $\frac{3}{4}+\frac{5}{6}$.
51. $\frac{7}{12}+\frac{5}{8}$.
52. $\frac{2}{3}+\frac{1}{6}$.
53. $\frac{7}{8}-\frac{1}{4}$.
54. $\frac{5}{6}+\frac{1}{2}$.
55. $\frac{7}{8}+\frac{3}{4}$.
56. $\frac{2}{3}-\frac{2}{7}$.
57. $\frac{8}{9}-\frac{5}{12}$.
58. $\frac{2}{9}+\frac{4}{5}$.
59. $2\frac{1}{2}+\frac{3}{4}+1\frac{3}{8}=?$
60. $4\frac{6}{7}+\frac{8}{9}+2\frac{4}{9}=?$
61. $17\frac{5}{12}+3\frac{7}{8}+19\frac{1}{6}=?$
62. $8\frac{1}{4}-7\frac{1}{8}=?$
63. $1\frac{1}{3}-\frac{2}{3}=?$
64. $205\frac{3}{4}-96\frac{5}{6}=?$
65. $8\frac{1}{3}+2\frac{1}{4}-3\frac{7}{12}=?$
66. $12\frac{1}{2}-8\frac{1}{3}+6\frac{1}{4}=?$

To multiply a fraction by an integer, either multiply the numerator of the fraction by the integer or divide the denominator by the integer. If the denominator contains the integer an integral number of times it is better to divide the denominator. Why?

The product of two or more fractions is the product of their numerators divided by the product of their denominators. Before finding these products cancel their common factors.

67. Find the product of $8\frac{1}{3}$, $\frac{4}{15}$, and $4\frac{1}{5}$.

SOLUTION. $$8\tfrac{1}{3}\times\tfrac{4}{15}\times4\tfrac{1}{5}=\frac{\overset{\cancel{5}}{\cancel{25}}}{3}\times\frac{4}{\underset{\cancel{3}}{\cancel{15}}}\times\frac{\overset{7}{\cancel{21}}}{\cancel{5}}=\frac{28}{3}=9\tfrac{1}{3}.$$

Write only the products of these fractions :

68. $\frac{1}{6}\times3$; $\frac{7}{12}\times4$; $\frac{5}{9}\times3$; $\frac{2}{3}\times\frac{5}{6}$; $6\times\frac{3}{5}$.

69. $\frac{8}{9}\times\frac{5}{12}$; $10\times\frac{7}{30}$; $\frac{5}{8}\times\frac{3}{10}$; $\frac{4}{9}\times\frac{3}{5}\times15$.

70. $6\times2\frac{1}{2}$; $3\frac{1}{3}\times\frac{2}{5}$; $6\frac{1}{4}\times8$; $16\frac{2}{3}\times6$.

71. $6\frac{1}{4}\times12$; $\frac{3}{5}\times12\frac{1}{2}$; $8\times37\frac{1}{2}$; $\frac{9}{40}\times8\frac{1}{3}$.

72. $2\times\frac{7}{8}\times\frac{5}{14}\times\frac{2}{15}$.

73. $2\frac{5}{8}\times1\frac{1}{7}\times1\frac{1}{3}\times5=?$

74. $3\frac{6}{7}\times\frac{5}{9}\times\frac{7}{24}=?$

To multiply a mixed number it is generally easier first to multiply the fraction of the mixed number, then the integer. Thus, $$\tfrac{5}{6}\times24\tfrac{2}{3}=\tfrac{5}{6}\times\tfrac{2}{3}+\tfrac{5}{6}\times24=\tfrac{5}{9}+20=20\tfrac{5}{9}.$$

75. Multiply $289\frac{7}{8}$ by 16; by $\frac{1}{2}$.

76. Find the cost of $376\frac{2}{3}$ tons of coal at \$3.25 a ton.

77. Find the cost of $156\frac{5}{12}$ acres of land at \$$116\frac{2}{3}$ an acre.

To divide a fraction by an integer either divide its numerator by the integer or multiply its denominator by the integer. When should the first plan be used and when the second?

Write these quotients:

78. $\frac{4}{5} \div 2$; $\frac{3}{7} \div 5$; $\frac{8}{9} \div 3$; $\frac{8}{9} \div 4$; $\frac{15}{16} \div 5$.

79. $1\frac{1}{2} \div 3$; $1\frac{1}{2} \div 2$; $\frac{7}{8} \div 4$; $2\frac{1}{4} \div 3$; $\frac{6}{7} \div 4$.

80. $12\frac{1}{2} \div 5$; $16\frac{2}{3} \div 20$; $37\frac{1}{2} \div 6$; $\frac{1}{2} \div 7$.

81. $83\frac{1}{3} \div 6$; $41\frac{2}{3} \div 25$; $6\frac{1}{4} \div 15$; $\frac{3}{4} \div 5$.

To divide by a fraction invert the divisor and proceed as in multiplication of fractions.

Find these quotients:

82. $\frac{8}{9} \div \frac{2}{3}$; $\frac{5}{12} \div \frac{3}{4}$; $\frac{5}{8} \div \frac{3}{4}$; $\frac{3}{10} \div \frac{5}{6}$.

83. $\frac{12}{25} \div \frac{3}{5}$; $\frac{14}{15} \div \frac{7}{30}$; $\frac{33}{40} \div \frac{11}{12}$; $5\frac{1}{7} \div \frac{3}{14}$.

84. $2 \div 1\frac{1}{2}$; $6\frac{1}{4} \div 2\frac{1}{2}$; $37\frac{1}{2} \div 12\frac{1}{2}$; $1 \div \frac{1}{3}$.

85. $50 \div 12\frac{1}{2}$; $3\frac{1}{5} \div 3\frac{1}{8}$; $10 \div 7\frac{1}{2}$; $8\frac{4}{5} \div 1\frac{3}{8}$.

A **complex fraction** is one which has a fraction in one or both of its terms.

A complex fraction can be reduced to a simple fraction by multiplying both of its terms by their least common denominator.

EXAMPLE. Reduce $\dfrac{2\frac{1}{2}}{3\frac{1}{3}}$ to a simple fraction.

SOLUTION. $\dfrac{2\frac{1}{2}}{3\frac{1}{3}} = \dfrac{15}{20}$, multiplying both terms by 6,

$= \dfrac{3}{4}$, reducing to lowest terms.

Reduce to simple fractions:

86. $\dfrac{\frac{5}{8}}{\frac{1}{4}}$. **87.** $\dfrac{\frac{7}{12}}{\frac{5}{6}}$. **88.** $\dfrac{\frac{1}{5}}{\frac{1}{3}}$. **89.** $\dfrac{\frac{2}{3}}{\frac{3}{4}}$. **90.** $\dfrac{\frac{5}{6}}{\frac{3}{4}}$. **91.** $\dfrac{\frac{7}{8}}{\frac{1}{6}}$. **92.** $\dfrac{\frac{9}{10}}{\frac{4}{15}}$.

93. $\dfrac{\frac{11}{12}}{2}$. **94.** $\dfrac{5}{\frac{2}{3}}$. **95.** $\dfrac{12}{\frac{6}{7}}$. **96.** $\dfrac{1\frac{1}{2}}{\frac{3}{4}}$. **97.** $\dfrac{1\frac{7}{8}}{\frac{5}{6}}$. **98.** $\dfrac{2\frac{1}{2}}{7\frac{1}{2}}$.

99. $\dfrac{50}{16\frac{2}{3}}$. **100.** $\dfrac{100}{6\frac{1}{4}}$. **101.** $\dfrac{12\frac{1}{2}}{100}$. **102.** $\dfrac{100}{33\frac{1}{3}}$. **103.** $\dfrac{100}{83\frac{1}{3}}$.

104. Simplify $\frac{4}{5}$ of $1\frac{7}{8} + 4\frac{2}{3}$.

105. Simplify $25 \div 8\frac{1}{3} - 2 \times \frac{2}{3} + 1\frac{5}{6}$.

Miscellaneous Formulas

16. Some new symbols. In the following exercises the pupil needs to know the meaning and use of certain symbols.

5^2 means 5×5 and is read "5 square."

a^2 means aa, and if $a=7$ then $a^2=7 \times 7=49$.

a' is read "a prime" and it is used to stand for a number different from a but used in the same way. Thus if we are making a formula in which the circumferences of two circles are used, we may represent the circumference of one of the circles by c, the circumference of the other by c'.

Parentheses around an expression mean that the processes indicated within the parentheses are to be performed first. Thus, $5(4+3)$ means that 4 and 3 are to be added, then their sum is to be multiplied by 5.

If an expression requires additions and subtractions only, these are performed in the order indicated. Thus, $12-7-2+5$ means that 7 is to be subtracted from 12, 2 from the result, and 5 added to that result. This same rule applies to expressions requiring only multiplications and divisions. If an expression requires additions or subtractions as well as multiplications or divisions, the multiplications and divisions are performed before the additions and subtractions, unless parentheses indicate that the additions or subtractions are to be performed first.

Thus, the expression $12 \times 2+4 \times 5-15 \div 3$ means that 12 is to be multiplied by 2, then 4 multiplied by 5, then 15 divided by 3; then the two products, 24 and 20, are to be added and the quotient 5 subtracted from their sum.

Example. In the formula $l=\frac{\pi}{2}(D+D')+2d$, find the value of l if $\pi=3\frac{1}{7}$, $D'=\frac{2}{3}$, $D=\frac{1}{2}$, and $d=\frac{3}{8}$.

Solution. Substituting the values of the letters in the formula,

$$l=\frac{3\frac{1}{7}}{2}\left(\frac{1}{2}+\frac{2}{3}\right)+2\times\frac{3}{8}=\frac{22}{14}\left(\frac{7}{6}\right)+\frac{3}{4}=\frac{11}{6}+\frac{3}{4}=2\frac{7}{12}.$$

Exercise 15

1. In the formula $P=\frac{wW}{p}$, find the value of P, if
   (a) $W=6$, $w=4$, and $p=16$;
   (b) $w=\frac{5}{12}$, $W=6$, and $p=2$;
   (c) $w=1\frac{4}{5}$, $W=\frac{5}{18}$, and $p=\frac{1}{4}$.

2. In the formula $h=2d+a$, find the value of h, if
   (a) $d=12\frac{7}{8}$ and $a=\frac{1}{4}$;
   (b) $a=5\frac{3}{4}$ and $d=3\frac{1}{3}$;
   (c) $d=\frac{3}{40}$ and $a=\frac{2}{25}$;
   (d) $a=50$ and $d=37\frac{1}{2}$.

3. In the formula $F=\frac{f+f'}{ff'}$ find the value of F, if
   (a) $f=3$ and $f'=4$;
   (b) $f=\frac{7}{8}$ and $f'=\frac{3}{4}$;
   (c) $f=2$ and $f'=4\frac{1}{5}$;
   (d) $f=25$ and $f'=\frac{3}{100}$.

4. Make a formula for finding the area, A, of a square whose side is s; also a formula for finding D, the difference of the areas of two squares whose sides are S and s.

5. Make a formula for finding the distance apart of two trains which started at the same station and ran for 6 hours in opposite directions at the rates r and r'.

6. Make a formula for finding the number of inches, i, in a ft. b in.

7. Make a formula for finding the number of inches, I, in y yd. f ft. i in.; the number of cents, C, in a dollars, b dimes, and c cents.

8. Make a formula for finding the total weight, W, of a bag weighing b ounces containing 15 marbles each weighing m ounces.

Exercise 16. Review

1. Multiply each of these numbers by 2 : 27, 19, 242, 526, 837, 1072.

2. Multiply by 6 : 18, 42, 75, 125, 820, 172.

3. Multiply by 9 : 7, 12, 24, 30, 502, 631.

Multiply :

4. 867 by 35. 5. 4865 by 297. 6. 7024 by 302.

7. $67\frac{1}{2}$ by $12\frac{1}{2}$. 8. 968 by $18\frac{3}{4}$. 9. $78\frac{3}{5}$ by $3\frac{1}{6}$.

10. 1001 by $33\frac{1}{3}$.

11. Divide by 2 : 26, 96, 150, 480, 300, 5022.

12. Divide by 3 : 21, 90, 132, 255, 801, 1242.

13. Divide by 8 : 48, 72, 104, 144, 320, 568.

Divide :

14. 4536 by 21. 15. 46,216 by 53. 16. 11,413,383 by 201.

17. 316,587 by 200. 18. 8324 by 144. 19. $612\frac{1}{2}$ by $2\frac{1}{3}$.

20. $84\frac{5}{6}$ by $7\frac{1}{2}$.

21. State the cost formula.

22. State the formula for finding the volume of a rectangular solid.

23. Write the sum of 3 and x ; write the difference when x is subtracted from 3 ; write the product of 3 and x ; write the quotient of x divided by 3 ; the quotient of 3 divided by x.

24. One line is a in. long, another b in., and a third c in. Find the sum of their lengths.

25. How many pecks in b bushels? In x bushels and 3 pecks?

26. A traveler starts from a certain point, A, and drives m miles east, then x miles west, then 24 miles east. How far *is he then from A?*

27. How many miles in r rods? How many rods in m miles?

28. A family uses $\frac{2}{3}$ of a sack of flour in a week. At that rate how long will it take them to use 4 sacks?

29. How many boards each $\frac{3}{8}$ in. thick will make a pile 4 ft. 3 in. high?

30. A man does a piece of work in 8 days. At that rate what part does he do in 1 day? In 5 days?

31. A does a piece of work in 6 days and B in 4 days. What part does each do in one day? What part do they both together do in 1 day? How long will it take both of them to do $\frac{1}{12}$ of the work? To do $\frac{12}{12}$ of the work?

32. John can mow a lawn in $2\frac{1}{2}$ hr., and William can mow it in 3 hr. What part can they both do in 1 hr.? How many hours will it take them to mow the lawn working together?

33. $V=\frac{mn}{m+n}$. Find V when $m=100$ and $n=65$; when $m=\frac{2}{3}$ and $n=\frac{5}{12}$.

34. $R=\frac{2s}{r}-\frac{r}{2s}$. Find R when $s=12\frac{1}{2}$ and $r=3\frac{1}{4}$.

35. Three circles each of diameter d and 2 circles each of diameter d' are placed in a row touching each other, and with their centers on the same straight line. Write the formula for finding the length of the row.

36. $Q=\frac{1}{m-a}-\frac{1}{n-b}$. Find Q when $a=\frac{1}{8}$, $b=\frac{1}{4}$, $m=\frac{2}{3}$, and $n=\frac{5}{6}$.

CHAPTER III

DECIMAL FRACTIONS AND THE METRIC SYSTEM

17. Reading and writing numbers. Any number may be written by using the ten figures 0, 1, 2, 3, 4, 5, 6, 7, 8, 9. Each figure has a form value and a place value. The figure 3 when standing alone or in the first place to the left of the decimal point has the form value three units. When it stands in the second place to the left of the decimal point its value is ten times its form value; in the third place, one hundred times its form value; and so on. When a figure stands in the first place to the right of the decimal point its value is one-tenth of its form value; in the second place, one-hundredth of its form value; and so on.

The places or **orders** to the left of the decimal point are called units, tens, hundreds, thousands, ten-thousands, hundred-thousands, millions, ten-millions, and so on.

For convenience in reading, the orders are named in groups of three, each group being called a **period**. The first six periods to the left of the decimal point are units, thousands, millions, billions, trillions, and quadrillions.

From the decimal point to the right the orders are named tenths, hundredths, thousandths, ten-thousandths, hundred-thousandths, millionths, ten-millionths, hundred-millionths, and so on.

The word *and* is used to connect an integer and a fraction, either common or decimal, but is not used in reading an *integer.*

EXAMPLES. 10,475,001 is read ten million four hundred seventy-five thousand one.

1080.02003 is read one thousand eighty and two thousand three hundred-thousandths.

$5.02\frac{2}{3}$ is read five and two and two-thirds hundredths.

$5.00\frac{2}{3}$ is read five and two-thirds of a hundredth.

Exercise 17

1. Name the following orders to the left of the decimal point: The first, the fourth, the seventh; the second, the fifth, the eighth; the third, the sixth, the ninth.

2. Give the number of each of the following orders, counting from the decimal point: Units, thousands, millions; tens, ten-thousands, ten-millions; hundreds, hundred-thousands, hundred-millions.

3. Give the number of each of the following orders and tell whether it is at the right or the left of the decimal point: Tens, hundredths, tenths, hundred-millionths, thousandths, ten-thousands.

Read the following numbers:

4. 687.
5. 2590.
6. 2004.
7. 12746.
8. 865032.
9. 3860000.
10. 25901004.
11. 312000962.
12. .35.
13. .003.
14. .567.
15. 4.3.
16. 8.605.
17. 3.1416.
18. 1.4142.
19. 1.732.
20. 86.0501.
21. .0043.
22. .000006.
23. .0103.
24. 10.3.
25. 1.03.
26. .230.
27. 2.30.
28. .2300.
29. .0230.
30. 23.0.
31. 807.01.
32. 80.701.
33. .08701.
34. 80.7010.
35. .80701.
36. 200,005.
37. 200.005.
38. .205.
39. 2.00005.

40. The following is a clipping from a daily paper. Read it. "It is estimated that of the approximately 5,000,000,000,000 board-feet of merchantable timber that was originally comprised in the forests of the United States about 2,700,000,000,000 remain. Of this there is owned privately 2,020,000,000,000; by the National Government and by the States, 680,000,000,000. While the annual consumption of lumber is at the rate of approximately 40,000,000,000, the new growth is at the rate of less than 20,000,000,000 board-feet."

Exercise 18

Write in figures the following numbers:

1. Twenty-three ten-thousandths.

2. One and two hundredths.

3. Seventy and one hundred one ten-thousandths.

4. Ninety billion eight million two hundred one thousand seven.

5. One million and two hundred-millionths.

6. One million two hundred-millionths.

7. One million and two hundred millionths.

8. Six and one-half tenths.

9. Six and one-half of a tenth.

10. Seven thousand two and two and one-third hundredths.

11. Three-fourths of a tenth.

12. Three-fourths of a hundredth.

13. Five hundred thousand five millionths.

14. Five hundred thousand and five hundred-thousandths.

15. In a recent year the value of the crops in the United States was estimated at five billion two hundred million *seventy-eight thousand* dollars.

18. Problems involving large numbers. In problems involving multiplications and divisions time and labor are saved by first indicating the multiplications and divisions and then performing them by cancellation.

EXAMPLE. The first Liberty Loan of the United States in the European War was \$2,000,000,000. A silver dollar is $1\frac{1}{2}$ in. in diameter. If 1,000,000,000 silver dollars were placed in a row just touching each other, they would make a row how many miles long? Neglect the fraction of a mile.

SOLUTION. The indicated multiplications and divisions are

$$\frac{1,000,000,000 \times 1.5}{5280 \times 12}.$$

Let the pupil complete the solution, canceling as far as possible.

Exercise 19

1. How many years and days will it take to count 1,000,000,000, counting at the rate of 150 a minute, and counting 8 hours a day?

2. In what year was an event that happened one billion minutes after the beginning of the Christian era?

3. If there are 3 birds to the acre in Illinois, and each bird destroys one-third of an ounce of insects a day, how many tons of insects are destroyed in 6 months of 30 days each? The area of Illinois is 56,650 square miles.

4. Light travels 186,000 miles per second. How long does it take light to travel from the sun to the earth, 93,000,000 miles?

5. Sound travels 1084 ft. per second. The distance from the earth to the moon is 240,000 miles. If you waved your handkerchief and at the same time said "Hello" to the "Man in the Moon," and if he received both signals, how long before he heard the hello would he see the handkerchief? *Neglect the fractions* of a second.

6. How many miles will light travel in a year of $365\frac{1}{4}$ days?

7. The distance that light travels in one year is called a light year. The nearest fixed star is said to be 3.6 light years distant from the earth. How many miles distant is it?

8. It is thought that some stars are as far away as 100,000 light years. How many miles distant are they?

9. The distance from the sun to the planet Neptune is 2,792,000,000 miles. Neptune is how many times as far away from the sun as the earth?

10. The annual yield of wheat in the United States for ten years was as follows :

634,087,000,	664,602,000,	683,379,000,	635,121,000,
621,338,000,	730,267,000,	763,380,000,	891,017,000,
1,011,505,000,	607,557,000.		

Find the average annual yield for these years.

11. The corn crop in the United States for a certain year was estimated to be three billion fifty-four million five hundred thirty-five thousand bushels, with a total value of one billion seven hundred fifty-five million eight hundred fifty-nine thousand dollars. What was the estimated price a bushel?

19. Reduction of decimal fractions to common fractions.

Example 1. Reduce .625 to a common fraction in its lowest terms.

Solution. $.625 = \frac{625}{1000} = \frac{5}{8}$.

Example 2. Reduce $.16\frac{2}{3}$ to a common fraction in its lowest terms.

Solution. $16\frac{2}{3} = \frac{16\frac{2}{3}}{100} = \frac{1}{6}$.

What are the steps in reducing decimal fractions to common fractions in their lowest terms?

Exercise 20

Reduce to common fractions in their lowest terms:

1. .75.	**6.** 67.	**11.** .125.	**16.** .0002.
2. .45.	**7.** .8.	**12.** .375.	**17.** .0096.
3. .16.	**8.** .05.	**13.** .025.	**18.** .525.
4. .52.	**9.** .008.	**14.** .875.	**19.** .1875.
5. .35.	**10.** .045.	**15.** .317.	**20.** .3125.

21. .0025. **22.** 5.25.

SOLUTION. $5.25 = 5\frac{25}{100}$
$= 5\frac{1}{4}$.

23. 19.125.	**28.** 904.375.	**33.** $.33\frac{1}{3}$.	**38.** $.016\frac{2}{3}$.
24. 78.5.	**29.** 3.875.	**34.** $.62\frac{1}{2}$.	**39.** $4.6\frac{1}{3}$.
25. 3.14.	**30.** $.12\frac{1}{2}$.	**35.** $.87\frac{1}{2}$.	**40.** $7.66\frac{2}{3}$.
26. 62.004.	**31.** $.37\frac{1}{2}$.	**36.** $.83\frac{1}{3}$.	**41.** $.60\frac{1}{3}$.
27. 436.75.	**32.** $.08\frac{1}{3}$.	**37.** $.03\frac{1}{3}$.	**42.** $1.01\frac{1}{2}$.

Rule. *To reduce a decimal fraction to a common fraction, write the numerator over the denominator and reduce the result to its lowest terms.*

20. Multiplication and division by powers of 10. The numbers 10, 100, 1000, and so on are called powers of 10.

In multiplying and dividing by powers of 10 the pupils should use the following rules :

To multiply a number by a power of 10 move the decimal point as many places to the right as there are zeros in the multiplier.

To divide a number by a power of 10 move the decimal point as many places to the left as there are zeros in the divisor.

Since $25 = \frac{100}{4}$, we can multiply by 25 by multiplying by 100 and dividing by 4. Also we can divide by 25 by dividing by 100 and multiplying by 4.

Thus, $25 \times 3.472 = \dfrac{100 \times 3.472}{4} = \dfrac{347.2}{4} = 86.8.$

Exercise 21

1. Copy and fill out the following table. Multiply the numbers in the left-hand column by the number at the top of each of the other columns, and write the results in the vacant spaces.

	10	100	1000	1,000,000
4.001				
385.6				
.01037				
$.008\frac{1}{3}$				
$.012\frac{1}{2}$				
$.0006\frac{2}{3}$				

2. Make a similar table and divide each of the numbers 423.7, 100.2, $.0\frac{1}{3}$, $42\frac{2}{3}$, 20.09, $\frac{1}{7}$ by each of the numbers 10, 100, 1000, 1,000,000.

3. Since $33\frac{1}{3}=\frac{100}{3}$, make rules for multiplying and for dividing by $33\frac{1}{3}$.

4. Since $125=\frac{1000}{8}$, make rules for multiplying and for dividing by 125.

5. What part of 100 is 50? Make rules for multiplying and for dividing by 50; by $16\frac{2}{3}$; by $12\frac{1}{2}$; by $8\frac{1}{3}$.

6. $47.584\times25=?$

7. $62.25\div25=?$

8. $486\times33\frac{1}{3}=?$

9. $963.4\div125=?$

10. $\$186.15\div33\frac{1}{3}=?$

11. $.0048\times125=?$

12. $54\times16\frac{2}{3}=?$

13. $42.08\times50=?$

14. $8\frac{1}{3}\times240=?$

15. $842.96\div16\frac{2}{3}=?$

16. $25\times128.464=?$

17. $125\times.0424=?$

18. $1000\div33\frac{1}{3}=?$

19. $84\times33\frac{1}{3}\div25=?$

21. Multiplication of decimal fractions. Principle. *The number of decimal places in the product of several decimals equals the sum of the numbers of decimal places in the factors.*

EXAMPLE. Multiply 24.74 by $.08\frac{1}{3}$.

SOLUTION.

$$\begin{array}{r} 24.74 \\ .08\frac{1}{3} \\ \hline 824\frac{2}{3} \\ 19792 \\ \hline 2.0616\frac{2}{3} \end{array}$$

Exercise 22

Find the following products :

1. $.8\times.03$.
2. $.46\times.002$.
3. $.309\times100$.
4. $.12\times.12$.
5. $.007\times200$.
6. 300×2.43.
7. $.1\times.2\times.3$.
8. $.07\times.11\times.01$.
9. 36×3.1416.
10. 27×1.4142.
11. 8.5×54.68.
12. 43.5×9.03.
13. $.03\frac{1}{3}\times.685$.
14. $.00\frac{1}{3}\times942$.
15. $2.01\frac{1}{6}\times1200$.
16. $.00\frac{1}{5}\times35.8$.

17. Gold is 19.3 times as heavy as water. A cubic foot of water weighs 62.5 lb. What is the weight of a cubic foot of gold?

18. Find the weight of a cubic foot of alcohol which is .79 as heavy as water.

19. A certain kind of steel bar weighs 3.83 lb. for each foot of length. Find the weight of such a bar 16 ft. long.

20. What is the cost of 450 bu. of wheat at $1.87 a bushel?

21. A certain type of steel bolt weighs .052 lb. each. Find the weight of 100 of these bolts ; of 2000 bolts.

22. Another type of bolt weighs .143 lb. each. Find the weight of 500 of these bolts.

23. What is the cost of $3\frac{1}{4}$ yards of cloth at $1.30 a yard?

22. Division of decimal fractions by integers. Give the following quotients:

$6÷2.	15 hundredths÷5.	.6÷2.	.0048÷8.
9 tenths÷3.	12 thousandths÷6.	.008÷4.	.65÷5.

In each case state the number of decimal places in the dividend and the number in the quotient.

These examples illustrate the following important **principle.** *When a decimal fraction is divided by an integer the number of decimal places in the quotient is the same as the number in the dividend.*

Exercise 23

1. .035÷7.	4. 45.6÷3.	7. 96.3÷100.	10. .0093÷30.
2. .65÷5.	5. 10.06÷2.	8. 27.27÷10.	11. .004÷200.
3. .0024÷8.	6. 607.5÷5.	9. .4÷20.	12. .05÷500.

13. Divide $10.1\frac{2}{3}$ by 10; by 100; by 1000.

14. According to the United States Life Tables, which are computed for a portion of the United States, of 100,000 persons born alive, the numbers reaching certain ages are given in the following table:

Age	10	20	30	40	50	60	70	80	90
Number living	82,458	80,074	75,779	70,042	62,460	51,138	33,816	13,712	1,868

Find the fraction of the 100,000 that is living at each of the given ages.

23. Reduction of common fractions to decimal fractions.

Example 1. Reduce $\frac{7}{25}$ to a decimal fraction.

Solution 1.
$$\begin{array}{r} .28 \\ 25\overline{)7.00} \\ \underline{50} \\ 200 \\ \underline{200} \end{array}$$

Solution 2. $\frac{7}{25}=\frac{4\times 7}{4\times 25}=\frac{28}{100}=.28.$

EXAMPLE 2. Reduce $\frac{1}{3}$ to thousandths.

SOLUTION.
$$3\overline{)1.000}\quad .333\tfrac{1}{3}$$

Can you make rules, corresponding to these two forms of solution, for reducing common fractions to decimal fractions?

Exercise 24

In the following exercises choose the form of solution that is more convenient.

Reduce to decimals :

1. $\frac{1}{2}$.	6. $\frac{1}{8}$.	11. $\frac{7}{8}$.	16. $\frac{5}{32}$.	21. $\frac{19}{2000}$.
2. $\frac{1}{4}$.	7. $\frac{3}{8}$.	12. $\frac{19}{50}$.	17. $\frac{38}{5}$.	22. $\frac{13}{16}$.
3. $\frac{3}{4}$.	8. $\frac{3}{20}$.	13. $\frac{7}{500}$.	18. $\frac{42}{125}$.	23. $\frac{7}{64}$.
4. $\frac{1}{5}$.	9. $\frac{23}{50}$.	14. $\frac{69}{20}$.	19. $\frac{62}{125}$.	24. $\frac{1}{80}$.
5. $\frac{4}{5}$.	10. $\frac{5}{8}$.	15. $\frac{1}{200}$.	20. $\frac{1}{1000}$.	25. $\frac{18}{45}$.

Reduce to thousandths :

26. $\frac{2}{3}$.	29. $\frac{5}{12}$.	32. $\frac{1}{45}$.	35. $\frac{12}{18}$.	37. $\dfrac{26}{8\frac{1}{3}}$.	39. $\dfrac{7\frac{2}{3}}{20}$.
27. $\frac{5}{6}$.	30. $\frac{1}{9}$.	33. $\frac{1}{270}$.	36. $\dfrac{20}{33\frac{1}{3}}$.	38. $\dfrac{11}{12\frac{1}{2}}$.	40. $\dfrac{435}{16\frac{2}{3}}$.
28. $\frac{1}{12}$.	31. $\frac{1}{30}$.	34. $\frac{11}{6}$.			

24. Division by decimals. Are the following quotients equal :

1. $9 \div 3 = 90 \div 30 = 900 \div 300$? 2. $8 \div 4 = 80 \div 40 = 8000 \div 4000$?

Supply the missing numbers in the following :

3. $12 \div 3 = 120 \div ?$
4. $45 \div 5 = 4500 \div ?$
5. $.016 \div 4 = ? \div 40$.
6. $.0018 \div 3 = .18 \div ?$
7. $64.09 \div 9 = ? \div 900$.
8. $.000035 \div 5 = ? \div 5000$.

These examples illustrate the principle :

The value of a quotient is not changed if the dividend and divisor are multiplied by the same number.

By the use of this principle any problem of division by a decimal may be changed to a problem of division by an integer. This last problem we know how to solve.

Example. Divide 6.856 by 3.14. Find the answer correct to .01.

Solution. The dividend and divisor are both multiplied by 100 so as to make the divisor an integer. The result of this multiplication is indicated by moving the decimal point two places to the right in both dividend and divisor. The crosses indicate the original places of the decimal points. The first figure in the quotient should be placed above the last figure in the dividend used in finding that figure of the quotient. The decimal point in the quotient should be placed over the new decimal point in the dividend.

$$\begin{array}{r} 2.18 \\ 3_{\times}14\overline{)6_{\times}85.6} \\ \underline{628} \\ 576 \\ \underline{314} \\ 2620 \\ \underline{2512} \\ 108 \end{array}$$

It has been found that a much higher degree of accuracy of results is obtained when the divisor is first made an integer. This method of division of decimals is fast coming into general use.

Exercise 25

Find the following quotients :

1. $.002 \div .02$.	9. $.00335 \div 6.7$.
2. $.0027 \div .01$.	10. $.05475 \div 3.5$.
3. $4.8 \div .4$.	11. $105.7 \div 3.5$.
4. $.245 \div .3$.	12. $.11928 \div .056$.
5. $3.1416 \div .06$.	13. $.04905 \div .327$.
6. $.48 \div .16$.	14. $68.275 \div 145$.
7. $.48 \div 16$.	15. $.33615 \div 12.45$.
8. $.0048 \div .16$.	16. $.014532 \div .0692$.

17. At $11.25 a ton how many tons of coal can be bought for $292.50?

18. There are 31.5 gallons in a barrel. How many barrels are there in 693 gallons?

25. Getting results to a desired degree of accuracy.

EXAMPLE. What is the cost of 5 yd. of cloth at $8\frac{1}{3}$¢ a yard?

The cost at this rate is $41\frac{2}{3}$¢. Since we have no coin of smaller value than 1¢, the cost is fixed to the nearest cent. In this case the cost is nearer 42¢ than 41¢, and the merchant charges 42¢.

In general when the fraction of a cent is one-half or more the next greater integral number is taken; if the fraction of a cent is less than one-half the next smaller number is taken. Thus, the three amounts $41\frac{1}{3}$¢, $41\frac{1}{2}$¢, and $41\frac{2}{3}$¢ taken to the nearest cent are 41¢, 42¢, and 42¢.

Exercise 26

1. Find the cost of 16 yd. of cloth at $8\frac{1}{3}$¢ a yard.

2. Find the cost of 4 dozen and 10 eggs at 25¢ a dozen.

3. Find the amount of a laundry bill for 7 collars at $2\frac{1}{2}$¢, 5 shirts at $12\frac{1}{2}$¢, and 4 handkerchiefs at 2¢.

4. At a sale, articles are marked to sell at $\frac{3}{4}$ of their usual selling price, to the nearest 5¢. For example, an article which usually sells for 75¢ is marked at 55¢. Find the new marked prices for articles which usually sell for \$1.25 ; 65¢ ; \$3.40 ; 80¢ ; \$1.85.

5. The following are the populations of 5 large cities of the United States according to the last census : 4,766,883 ; 2,185,283 ; 1,634,651 ; 1,549,008 ; 687,029. Read these numbers to the nearest 1000.

6. Read the following numbers correct to the nearest .01 : 7.846 ; 8.261 ; .049 ; .3557 ; .56 ; .07 ; .018. Read each correct to the nearest .1.

7. Find to the nearest 1000 the average population of the cities referred to in the fifth exercise.

8. *A load of coal* weighs 5435 lb. Find its cost at \$4.25 a *ton.*

9. To compare the standings of baseball teams ther computed for each team the fraction that the numbeı games it has won is of the total number of games it played. This result is usually computed to the neaı thousandth. If one team wins 77 games out of 95 and other team wins 74 out of 92, which has the higher standiı Compute the answers correct to thousandths.

SOLUTION.

$$\begin{array}{r} .8105 \\ 95\overline{)77.000} \\ \underline{760} \\ 100 \\ \underline{95} \\ 500 \\ \underline{475} \end{array} \qquad\qquad \begin{array}{r} .8043 \\ 92\overline{)74.000} \\ \underline{736} \\ 400 \\ \underline{368} \\ 320 \\ \underline{276} \end{array}$$

It is necessary to compute the number of ten-thousandths to the result correct to the nearest thousandth. Since 5 ten-tl sandths are one-half of 1 thousandth, if the number of ten-tl sandths is 5 or more, one is added to the thousandths; if the nun of ten-thousandths is less than 5, the ten-thousandths are dropı To the nearest thousandth the score of the first team is .811, of the second team, .804. The first team has the higher standiı

10. In a certain year the teams in the American Lea played and won the following numbers of games. Comp the fraction of games won by each team correct to .001.

TEAM	PLAYED	WON
Boston	151	101
Detroit	154	100
Chicago	154	93
Washington	153	85
New York	152	69
St. Louis	154	63
Cleveland	152	57
Philadelphia	152	43

11. The batting average of a baseball player is found by dividing the number of hits that he makes by the number of times he is at bat. The records made up to August in a certain year by certain of the best batters in the American and National Leagues are here given. Find the batting average of each, correct to .001.

PLAYER	TIMES AT BAT	HITS
Cobb	437	168
Speaker	416	145
Sisler	437	152
Roush	399	139
Kauff	384	124
Cruise	383	122

12. Forty-two acres of a farm of 160 acres are woodland. What part is woodland? Find the answer correct to .001.

13. On October 16, 1916, the national debt of the United States was estimated to be \$12 per capita, that is, \$12 for each person; that of Great Britain, \$376 per capita; of France, \$360; and of Germany, \$290. The per capita debt of each of the other countries was how many times that of the United States? Answer correct to .01.

14. Light travels in waves. The length of a wave of a certain kind of light is .0000154 in. How many such waves in a ray of this light 1 yd. long? Find result to the nearest unit.

15. A certain bacterium is .00001 in. long. How many such bacteria can be placed in a row an inch long?

16. A one-inch cube of metal is heated until its edges are each 1.003 in. Find its volume correct to the nearest .001 *cu. in. Find its total* surface correct to .001 sq. in.

17. Some ladies' suits are marked at the following prices: \$50 ; \$45.50 ; \$35 ; \$26.50 ; \$34.50 ; \$15.75 ; \$32.25 ; \$38 ; \$18 ; \$20.50 ; \$27.25 ; \$31.50. At a clearance sale they are all marked at the same price, which is $\frac{2}{3}$ of their average price, to the nearest 25¢. Find the sale price.

18. In the formula $F=\frac{f+f'}{ff'}$, find F, correct to the nearest .001, when

(*a*) $f=.02, f'=50$.
(*b*) $f=.75, f'=.24$.
(*c*) $f=2, f'=.0003$.
(*d*) $f=38.1, f'=17.56$.

19. A match is .1 in. square on the end and $2\frac{1}{4}$ in. long. Find the volume of 10,000 matches.

Find the following quotients :

20. $4 \div .1$.	**23.** $369 \div .009$.	**26.** $57 \div 200$.
21. $57 \div .01$.	**24.** $2 \div .005$.	**27.** $.6 \div 150$.
22. $68.4 \div .02$.	**25.** $14.75 \div .0025$.	**28.** $.02 \div 40$.

Find the following quotients correct to .001 :

29. $8\frac{1}{3} \div 4$.	**31.** $25.4 \div 1.732$.	**33.** $87\frac{1}{2} \div 1.414$.
30. $.06\frac{1}{7} \div 5$.	**32.** $66\frac{2}{3} \div 19$.	**34.** $1 \div 3.1416$.

35. The circumference of a circle is 3.1416 times the diameter. In a piece of machinery it is desired to have the circumference of a wheel exactly 10 ft. Find the diameter correct to .01 in.

36. The diagonal of a square is 1.4142 times the side. Find the diagonal of a field which is 40 rods square, correct to .01 of a rod.

37. Twelve measurements of the diameter of a steel wire gave the following fractions of an inch : .061, .062, .064, .060, .061, .065, .059, .066, .058, .062, .060, .064. Find the average of these measurements to the nearest .001 in.

The Metric System

26. Advantages of the Metric System. The table of United States money is more easily learned than the table of linear measure because throughout the table of United States money the same number of units of any order makes one of the next higher.

In making the table of United States money 10 units of one order were taken to make one of the next higher, just as in ordinary numbers. This makes reduction from one denomination to another very simple. Thus, cents are reduced to a fraction of a dollar by dividing by 100, and may be written as a decimal. United States money and ordinary numbers are said to be written on a **scale** of 10, or on a **decimal scale.**

In computing weights and measures much time and labor are saved when all the tables are written on the scale of 10. Such a system of measures was devised by the French Government and is called the *Metric System.*

The convenience of the Metric System has led to its general adoption in Central and South America and in Europe except in Great Britain. It is used in science throughout the world. Its use in the United States would greatly simplify the learning and use of tables of weights and measures. The increased use of the Metric System in our foreign trade and the frequent use of metric units in articles in newspapers and magazines make it necessary for all Americans to become familiar with this system and its uses.

27. Linear measure. The **meter** is the primary unit of length of the metric system. All other units are based upon the meter. The meter was computed to be one ten-millionth of the distance from the equator to the pole. It is about 39.37 inches.

Decimal parts of the principal metric units are expressed

by Latin prefixes; multiples are expressed by Greek prefixes.

Latin Prefixes	Greek Prefixes
Milli means .001.	Deka means 10.
Centi means .01.	Hekto means 100.
Deci means .1.	Kilo means 1000.
	Myria means 10,000.

The following is the table of linear measure. In this and the following tables the names of the units commonly used are in black type.

Table of Linear Measure

10 millimeters (mm.) = 1 **centimeter** (cm.).
10 centimeters = 1 decimeter (dm.).
10 decimeters = 1 **meter** (m.).
10 meters = 1 dekameter (Dm.).
10 dekameters = 1 hektometer (Hm.).
10 hektometers = 1 **kilometer** (Km.).
10 kilometers = 1 myriameter (Mm.).

Lengths that are expressed in English units in inches, feet, and yards, and miles, are expressed in metric units in centimeters and millimeters, meters, and kilometers, respectively.

Note. The metric system cannot be taught successfully without having the metric units in the hands of the pupils. The pupils should do some measuring with these units. There should be in the schoolroom a meter stick, a liter measure, a decimeter cube, a number of centimeter cubes, balances, and a set of metric weights.

Exercise 27

1. Give the meaning of centi, milli, kilo, deci, hekto, deka, myria.

2. Give the prefix that means .001, 1000, .1, .01, 10,000, 100, 10.

3. A meter equals how many centimeters? How many decimeters? How many millimeters?

4. A *kilometer* equals how many meters?

5. What metric unit would be used in stating the length of a lead pencil? In stating the distance from New York to Chicago? The length of your schoolroom? The thickness of a telephone wire?

6. How far apart are the longest divisions on a meter stick? The next longest? The shortest?

7. How many decimeters in 45 m.? How many centimeters? How many millimeters?

8. Reduce 3657 mm. to meters; to centimeters; to decimeters.

9. What change in the position of the decimal point must be made in reducing centimeters to meters? Meters to kilometers? Meters to millimeters?

10. Reduce 2.75 cm. to meters.

11. Reduce .0028 m. to millimeters.

12. Reduce 486.7 meters to kilometers.

13. Reduce $3\frac{1}{4}$ m. to centimeters.

14. Reduce .117 Km. to meters.

15. Reduce 5.66 dm. to meters.

16. A kilometer is how many inches? Show that a kilometer is approximately $\frac{5}{8}$ of a mile, and that a centimeter is approximately $\frac{2}{5}$ of an inch. Use these equivalents in the next six exercises.

17. A mile is approximately how many kilometers?

18. An inch is approximately how many centimeters?

19. A 75 mm. gun is how many inches in diameter?

20. A trench is 120 Km. long. It is how many miles long?

21. A book is 8.5 cm. thick. Express the thickness in *inches.*

22. Measure the length of your desk in inches. Reduce to centimeters. Check your answer by measuring with a metric ruler.

23. The side of a square is 1 m. That is how many decimeters? The area of this square is how many square meters? How many square decimeters?

24. How many square decimeters make 1 square meter? How many square centimeters make 1 square decimeter? How many square units of one denomination make one of the next higher in the metric system?

25. The edge of a cube is 1 m. Each edge is how many decimeters? How many cubic decimeters in 1 cubic meter? How many cubic units of one denomination make one of the next higher in the metric system?

26. On the blackboard mark two points that you think are 1 m. apart, also two that you think are 1 dm. apart, and two that you think are 1 cm. apart. Test your estimates by measuring.

27. Hold your hand 1 m. from the floor. Let another pupil test your estimate by measuring.

28. Let each pupil measure the length of this page in centimeters. Let the results be written on the blackboard. Find the average of these measurements. Which result differs least from the average?

29. Let each pupil measure the length of this page in inches. Find the average. Reduce this average to centimeters. How does the result compare with the average found in the preceding problem?

30. Estimate certain distances in metric units, such as the length of the school yard, the distance from your home to school, the distance from New York to Chicago. Check these estimates when possible.

28. Square measure. A square meter is 10 decimeters on an edge, and therefore contains 10×10, or 100, square decimeters. Thus 100 square units of one denomination are required to make one of the next higher.

TABLE OF SQUARE MEASURE

100 square millimeters (mm.^{2}) =	1 **square centimeter** (cm.^{2}).
100 square centimeters =	1 square decimeter (dm.^{2}).
100 square decimeters =	1 **square meter** (m.^{2}).
100 square meters =	1 square dekameter (Dm.^{2}).
100 square dekameters =	1 square hektometer (Hm.^{2}).
100 square hektometers =	1 **square kilometer** (Km.^{2})

When used in measuring land a square meter is called a centare (ca.), a square dekameter an are (a.), and a square hektometer a hectare (Ha.).

29. Cubic measure. A cubic meter is 10 decimeters on each edge, and hence contains $10 \times 10 \times 10$, or 1000, cubic decimeters. Thus 1000 cubic units of one denomination are required to make one of the next higher.

TABLE OF CUBIC MEASURE

1000 cubic millimeters (mm.^{3}) =	1 **cubic centimeter** (cm.^{3}).
1000 cubic centimeters =	1 cubic decimeter (dm.^{3}).
1000 cubic decimeters =	1 **cubic meter** (m.^{3}).

Exercise 28

1. In the metric system how many linear units make one of the next higher denomination? How many square units? How many cubic units?

2. Measure a square meter on the blackboard and divide it into square decimeters.

3. On a sheet of paper mark a square decimeter and divide it into square centimeters.

4. Is your schoolroom large enough so that a square dekameter can be marked on the floor?

5. Reduce to square meters: 600 dm.^{2}; 18.56 Dm.^{2}; 200.2 cm.^{2}.

6. Read as square centimeters: 1.07 dm.^{2}; 978.5 mm.^{2}; .0025 m.^{2}.

7. Read as cubic decimeters: 1.5 m.^{3}; 493.75 cm.^{3}; 63,582 mm.^{3}.

30. Capacity and weight. The principal unit of capacity is the **liter** (lē-ter). It has the same volume as the cubic decimeter.

TABLE OF CAPACITY

10 milliliters (ml.) = 1 **centiliter** (cl.).
10 centiliters = 1 deciliter (dl.).
10 deciliters = 1 **liter** (l.).
10 liters = 1 dekaliter (Dl.).
10 dekaliters = 1 **hektoliter** (Hl.).

A liter is a little more than 1 liquid quart.

The principal unit of weight is the **gram**. It is the weight, under certain standard conditions, of 1 cm.^{3} of water.

TABLE OF WEIGHT

10 milligrams (mg.) = 1 centigram (cg.).
10 centigrams = 1 decigram (dg.).
10 decigrams = 1 **gram** (g.).
10 grams = 1 dekagram (Dg.).
10 dekagrams = 1 hektogram (Hg.).
10 hektograms = 1 **kilogram** (Kg.).
10 kilograms = 1 myriagram (Mg.).
10 myriagrams = 1 quintal (Q.).
10 quintals = 1 **metric ton** (T.).

The kilogram is used for weights which would be expressed in pounds in the English system. A kilogram is 2.2 lb., approximately.

Exercise 29

1. State how many liters in each of the other units in the table of capacity.

2. State how many grams in each of the other units in the table of weight.

3. Reduce to liters : 256 cl. ; 72.34 Hl. ; 205.5 dl.

4. Reduce to grams : 5298 mg. ; $6\frac{1}{3}$ Kg. ; $18\frac{3}{4}$ Kg.

5. What is the weight of 1 cm.^{3} of water? Of 250 cm.^{3} of water? Of 1000 cm.^{3}?

6. How many cubic centimeters in 1 dm.^{3}? What is the weight of 1 dm.^{3} of water?

7. How many cubic centimeters in one liter? What is the weight of one liter of water?

8. A vessel contains 12 l. of water. That amount of water weighs how many kilograms?

9. How many cubic decimeters in a solid 4 dm. by 3 dm. by 6 dm.?

10. A vessel is 6 dm. by 5 dm. by 2 dm., inside measurement. What is its capacity in liters? It is filled with water. How much does the water weigh?

11. A 2-gallon bucket holds about how many liters?

31. Equivalents. The pupils should memorize the following equivalents :

METRIC EQUIVALENTS	APPROXIMATE COMMON EQUIVALENTS
1 dm.^{3} = 1 l.	1 m. = 39.37 in.
The weight of 1 cm.^{3} of water = 1 g.	1 cm. = $\frac{2}{5}$ in.
	1 Km. = $\frac{5}{8}$ mi.
The weight of 1 dm.^{3} of water = 1 Kg.	1 hectare = $2\frac{1}{2}$ acres.
	1 l. = 1.1 liquid quarts.
The weight of 1 m.^{3} of water = 1 metric ton.	1 Kg. = 2.2 lb.
	1 metric ton = 2200 lb.

The following more exact equivalents should be used when solving problems unless stated otherwise:

1 Km. = .62138 mi.	1 g. = 15.432 grains.
1 m. = 39.37 in.	1 Kg. = 2.205 lb.
1 l. = 1.0567 liquid qt.	1 T. = 2204.62 lb.
= .908 dry qt.	1 $m.^3$ = 35.314 cu. ft.

1 hectare = 2.472 A.

Exercise 30

1. Read .042 m. as centimeters ; 74,286 mm. as meters ; 27 m. as kilometers.

2. Read 427 g. as kilograms ; .03 Kg. as grams.

3. Reduce 2 meters to yards.

4. Reduce 20 in. to centimeters.

5. Reduce 345 g. to pounds.

6. Reduce $3\frac{1}{3}$ l. to gallons.

7. Find the weight of 26.075 l. of water in grams ; in kilograms.

8. Using $2\frac{1}{2}$ cm. as 1 in., find the weight of a cu. in. of water in grams.

9. Using the result of the preceding exercise, find the weight of a cubic foot of water in grams ; in kilograms.

10. Knowing that 39.37 in. = 1 m., show that 1 in. = about $2\frac{1}{2}$ cm.

11. Show that 1 mi. = about $1\frac{3}{5}$ Km.

12. Knowing that 1 m. = 39.37 in., show that 1 $dm.^3$ = 61 cu. in., very nearly.

13. Gold is 19.26 times as heavy as water. How many grams does a cubic centimeter of gold weigh? How many kilograms does a cubic decimeter of gold weigh?

14. An American in Paris wishes to buy some collars and some shirts. His collars are $15\frac{1}{4}$ inches and his shirts have a 15-inch neckband and a 32-inch sleeve. What sizes shall he ask for in centimeters?

15. He must also buy a tire for his American automobile, which takes a 44-inch tire. What size shall he ask for in centimeters?

In the following exercises use the approximate equivalents:

16. A signboard in France gives the distance to the next town as 12 kilometers. How many miles is it?

17. At Verdun on a certain day the French army made a gain of 1300 meters on a battle front of 4 kilometers. Give these distances in miles.

18. The range of the German 42-centimeter gun is about 35 kilometers. Give the diameter of the gun bore in inches and its range in miles.

19. The speedometer on an American automobile in France shows that the car has gone 42.3 miles when the actual distance has been 66 kilometers. How much in error is the speedometer?

20. A brick is 25 cm. by 10 cm. by 6 cm. Find its volume in cubic decimeters.

21. A rectangular tank is 4 m. long and 85 cm. wide. The water in it is .5 m. deep. How many liters of water are in it?

22. Ice is .92 as heavy as water. What is the weight in kilograms of 1 $m.^3$ of ice?

23. What is the weight in pounds of 1 cu. yd. of ice? A cubic foot of water weighs 62.5 lb.

24. Which of the last two preceding exercises requires the less computation? Why?

25. How many gallons of water will a tank 10 ft. by 3 ft. by 2 ft. hold? (There are 231 cu. in. in 1 gal.)

26. How many liters of water will a tank 3 m. by 1 m. by 8 dm. hold?

27. Which of the last two preceding exercises requires the less computation? Why?

28. The Eiffel Tower in Paris is 300 m. high. How many feet high to the nearest foot?

29. The distance from Rome to Naples is 155 miles. How many kilometers?

30. The distance from Brussels to Ypres in Belgium is 122 Km. That is how many miles, to the nearest .1 mile?

Exercise 31. Review

1. Practice until you can write the last 24 sums on page 9 in 2 minutes or less.

2. Name and spell the names of the first six orders at the left of the decimal point.

3. Name and spell the names of the first six orders at the right of the decimal point.

4. Name and spell the names of the first six periods.

5. How is the average of several numbers found?

6. The daily attendance in a certain seventh grade for three weeks was as follows: 34, 36, 40, 40, 39, 38, 40, 38, 37, 38, 36, 37, 39. Find the average daily attendance. The number enrolled in this grade was 42. The average daily attendance was what part of the total enrollment? Find the answer correct to .001.

7. Find what part the average daily attendance for three weeks in your school is of the total enrollment.

8. Let each of 4 pupils measure the length of the schoolroom to the nearest centimeter. Find the average of these 4 results. Find how much each result differs from the average.

9. How many square millimeters in .6 $cm.^2$?

10. A man takes 240 steps in walking 180 meters. What is the average length of his steps?

11. One hektoliter of olive oil weighs how many kilograms, if olive oil is .92 times as heavy as an equal volume of water?

12. A box which is 1 m. long, 4.2 dm. wide, and 15 cm. deep, inside measurements, has a capacity of how many cubic centimeters?

13. Practice until you can find the differences on page 6 in 40 seconds or less.

14. A load of shelled corn weighs 3135 lb. What is it worth at 88 cents a bushel, 56 lb. to the bushel?

15. A 32-gallon barrel holds how many liters to the nearest liter?

16. State relations that exist between units of volume, weight, and capacity in the metric system.

17. $\dfrac{.15 \text{ of } 180+\frac{2}{3} \text{ of } 1.86}{\frac{3}{4}\div\frac{5}{6}}=?$ 18. $\dfrac{\frac{1}{2}+\frac{1}{3}+\frac{1}{4}}{2+3+4}=?$

19. Practice until you can find the products on page 6 in 55 seconds or less.

20. The following is a standard daily ration for United States troops :

Beef	2.00 oz.	Salt	.64 oz.
Flour	18.00 oz.	Pepper	.04 oz.
Baking powder . .	.08 oz.	Cinnamon	.014 oz.
Beans	2.40 oz.	Lard	.64 oz.
Potatoes	20.00 oz.	Butter	.50 oz.
Prunes	1.28 oz.	Sirup	.32 gill
Coffee	1.12	[illegible] .	.16 gill
Sugar	3 [illegible]	[illegible]	.014 gill
Milk, *evaporated* .	[illegible]		

Find the number of tons of each of the first 14 articles and the number of barrels of each of the last 3, required daily for 2,000,000 men.

21. A farmer has 7 bales of cotton averaging 495 lb. to the bale. He is offered 11¢ a pound for it, but decides to hold it for 6 months. During this time he pays 20¢ a bale each month for storage, and also $17 for insurance. At the end of the six months he sells the cotton for $12\frac{1}{4}$¢ a pound. Find the gain or loss if the cotton has lost in weight 7 lb. per bale.

22. A and B market their apples. A lives $\frac{1}{2}$ mile from market. Three men and two teams can haul and deliver 12 loads of 20 barrels each in a day. B lives 5 miles from market and must haul over poor roads. Three men and two teams can haul and deliver for him 4 loads of 15 barrels each in a day. The wages for a man are $2 a day and for a team $1.50 a day. How much more a barrel does it cost B than A to market his apples?

23. If $W=\frac{99900\,tbd}{l}$, find W when $t=.065$, $b=12$, $d=1.8$, and $l=48$.

24. Does $\frac{a}{c}+\frac{b}{c}=\frac{a+b}{c}$, when $a=.16$, $b=.012$, and $c=.003$?

25. Does $\frac{a}{b}+\frac{a}{c}=\frac{a}{b+c}$, when $a=1.85$, $b=\frac{2}{5}$, and $c=\frac{1}{8}$?

26. Does $\frac{a}{b}\times\frac{c}{d}=\frac{ac}{bd}$, when $a=.2$, $b=2.1$, $c=12$, and $d=.015$? When $a=840$, $b=.05$, $c=0$, and $d=16$?

CHAPTER IV

PERCENTAGE

32. The meaning of the term per cent.

1. A farmer pays as rent to the land owner 2 bushels out of every 6 bushels raised. What part of the crop is paid for rent? What part does the farmer keep?

2. Another farmer pays 2 bushels out of every 5 as rent. What part does he pay as rent? What part does he keep? Out of every 100 bushels of wheat raised, how much is given for rent and how much is kept by the farmer?

3. A boy put some money into the savings bank. At the end of a year the bank paid him as interest $3 for every $100 which he put into the bank. What part of the money put into the bank did the boy receive as interest? If he put $200 into the bank, how much interest did he receive?

4. In an examination 3 out of a class of 20 failed. What part of the class failed? What part of them passed? At that rate how many out of a hundred would fail?

5. In a class of 20, 7 failed, and in another class of 25, 8 failed. What part of each class failed? How many hundredths of each class failed? In which class did the larger part fail?

6. In a rifle match each contestant fired 100 shots. A hit the mark 85 times; B, 89 times; C, 91 times; and D, 78 times. How many hundredths of a perfect score did each make?

7. In a second match of 25 shots, A hit the mark 22 times; B, 20 times; C, 24 times; and D, 21 times. How many hundredths of a perfect score did each make? In which match did A make his better score? B? C? D?

8. In an examination the pupils are expected to answer 10 questions. John answers 9 questions correctly. What part does he answer correctly? How many hundredths?

9. In an examination with 5 questions 4 of them are answered correctly. What part is answered correctly? How many hundredths?

If $3\frac{1}{2}$ questions are answered correctly, what part is answered correctly? How many hundredths?

10. What fraction of a group is taken if we take

(*a*) 2 out of 7? (*c*) 9 out of 100? (*e*) 91 out of a hundred?
(*b*) 7 out of 50? (*d*) 47 out of 100? (*f*) 42 out of 100?

11. How many hundredths of a group are taken if we take

(*a*) 11 out of 50?	(*d*) 23 out of 25?	(*g*) 9 out of 60?
(*b*) 37 out of 50?	(*e*) 9 out of 20?	(*h*) 36 out of 80?
(*c*) 8 out of 25?	(*f*) 12 out of 40?	(*i*) 27 out of 90?

33. Summary. In computing a pupil's grade on an examination, the standing of a baseball team in a league, and in many business problems it is found convenient to compute according to a certain number out of a hundred. In comparing the scores in examples 6, 7, 8, and 9 it is convenient to have all the scores written as a number of hundredths. But instead of saying so many hundredths, or so many out of a hundred, or so many per hundred, we usually say so many **per cent.** The expression *per cent* is a short form of the Latin phrase *per centum,* which means by the hundred. The symbol % stands for the words per cent. Thus we write 3% for three per cent.

Since 1% means 1 out of a hundred,

$$1\% = .01 = \tfrac{1}{100}.$$

These three forms mean exactly the same thing. Similarly,

$$8\% = .08 = \tfrac{8}{100}, \text{ and } 125\% = 1.25 = \tfrac{125}{100}.$$

Exercise 32

1. How many hundredths of 100 is 40? How many per cent?

2. How many hundredths of 100 is each of the following numbers: 6, 18, 16, $12\frac{1}{2}$, $2\frac{1}{7}$, $8\frac{1}{3}$, 100, 120, 500, 842, 1234?

3. Tell how many per cent each is of 100.

4. A man has \$100 and gives \$6 to his son. What per cent of the money does the son get? What per cent does the father keep? If the son should receive the same per cent of \$200, how much would he receive?

5. In example 4, page 63, what per cent of the class failed? What per cent passed?

34. **Changing per cents to decimals.** Eight per cent, or 8%, means 8 hundredths and therefore 8% may be written .08, or $\frac{8}{100}$. The fraction $\frac{8}{100}$ may be written in its lowest terms $\frac{2}{25}$.

$$8\% = .08 = \frac{8}{100} = \frac{2}{25}.$$

Exercise 33

Express as decimals, then as common fractions and reduce to lowest terms the following:

1. 25%.	**8**. 6 %.	**15**. 440%.	**22**. $37\frac{1}{2}$%.	**29**. 2.5%.
2. 50%.	**9**. 95%.	**16**. 405%.	**23**. $62\frac{1}{2}$%.	**30**. 1.25%.
3. 75%.	**10**. 45%.	**17**. $33\frac{1}{3}$%.	**24**. $87\frac{1}{2}$%.	**31**. 1250%.
4. 40%.	**11**. 56%.	**18**. $66\frac{2}{3}$%.	**25**. $\frac{1}{3}$%.	**32**. 1000%.
5. 60%.	**12**. 234%.	**19**. $2\frac{1}{2}$%.	**26**. $\frac{5}{7}$%.	**33**. $133\frac{1}{3}$%.
6. 80%.	**13**. 547%.	**20**. $12\frac{1}{2}$%.	**27**. .6%.	**34**. .1%.
7. 22%.	**14**. 200%.	**21**. $5\frac{1}{3}$%.	**28**. .08%.	**35**. .75%.

Rule. *To change a per cent to a decimal remove the per cent sign and move the decimal point two places to the left.*

35. Important equivalents. The following equivalents are very important and should be memorized:

$50\% = \frac{1}{2}$.	$62\frac{1}{2}\% = \frac{5}{8}$.	$14\frac{2}{7}\% = \frac{1}{7}$.
$25\% = \frac{1}{4}$.	$87\frac{1}{2}\% = \frac{7}{8}$.	$10\% = \frac{1}{10}$.
$75\% = \frac{3}{4}$.	$16\frac{2}{3}\% = \frac{1}{6}$.	$20\% = \frac{1}{5}$.
$33\frac{1}{3}\% = \frac{1}{3}$.	$83\frac{1}{3}\% = \frac{5}{6}$.	$40\% = \frac{2}{5}$.
$66\frac{2}{3}\% = \frac{2}{3}$.	$8\frac{1}{3}\% = \frac{1}{12}$.	$60\% = \frac{3}{5}$.
$12\frac{1}{2}\% = \frac{1}{8}$.	$6\frac{1}{4}\% = \frac{1}{16}$.	$80\% = \frac{4}{5}$.
$37\frac{1}{2}\% = \frac{3}{8}$.		

Exercise 34

1. Cover the right-hand side of the above equalities and see if you can give them in 20 seconds.

2. Cover the left-hand side of the equalities and see if you can give them in 20 seconds.

3. 50% of a bushel means what part of it?

4. 25% of 36 apples means what part of 36 apples?

5. $33\frac{1}{3}\%$ of the pupils of the seventh grade were absent one day. What part of them were absent? What part were present? What per cent were present?

6. $83\frac{1}{3}\%$ of the pupils in school have been vaccinated. What part have been vaccinated? What part have not? What per cent have not?

7. In a certain year Iowa had half of a full corn crop. What per cent of a full crop did the State have?

8. Of 20 words in a spelling test Harry missed 8. What part of the words did he miss? What per cent of them?

9. Mary solved 4 problems out of 5 correctly. What per cent did she solve correctly? What per cent did she miss?

10. From 2 gallons of whole milk a farmer got a pint of *cream.* What part of the whole milk was cream? What *per cent of it?*

11. A man planted 30 acres of corn. Three-fourths of the kernels germinated. What per cent of his crop did the man lose because the seed was poor?

12. One-sixth of a farmer's seed corn failed to germinate. What per cent germinated?

13. A farmer examining an ear of corn found that $\frac{1}{8}$ of the length of the ear was not filled out with kernels. What per cent of the length of the ear was filled out?

14. One-third of the population of a certain city is foreign born. What per cent is native?

15. In the world series baseball game between Brooklyn and Boston, the latter won 4 games and Brooklyn won 1. What part of the games did Boston win? What part did Brooklyn win? What per cent did each win?

16. A teacher sent 50% of her pupils to the playground, 25% of them to the library, and kept the remainder to assist her in putting the room in order. What part of her pupils were sent to the library? What part were kept to assist her? If she had 36 pupils, how many were in each of the three groups?

17. In making a journey a man rode 80 miles by train, then rode 30 miles by automobile, and then drove 10 miles. What part of his journey did he make by automobile? What part did he drive? What per cent of his journey did he make in each way?

18. A recipe for chocolate caramels calls for 1 cup of grated chocolate, 1 cup of brown sugar, $\frac{1}{2}$ cup of molasses, and $\frac{1}{2}$ cup of sweet milk. What part of the mixture is sweet milk? What part is chocolate? What per cent of each?

19. To make raspberry pastils the recipe requires $\frac{1}{2}$ of an ounce of raspberry juice, and $3\frac{1}{2}$ ounces of sugar. What part of the mixture is raspberry juice? What part is sugar? What per cent of each?

20. When the housewife buys a round steak she loses about $8\frac{1}{3}\%$ of it in bone and cutting. When she buys porterhouse steak she loses about $12\frac{1}{2}\%$ in bone and cutting. What part of her meat purchase is waste in each case?

Find

21. 50% of 180.
22. $37\frac{1}{2}\%$ of 248.
23. 25% of 48.
24. $62\frac{1}{2}\%$ of 176.
25. $8\frac{1}{3}\%$ of 72.
26. $87\frac{1}{2}\%$ of 32.
27. $33\frac{1}{3}\%$ of 156.
28. 60% of 75.
29. $12\frac{1}{2}\%$ of 424.
30. $16\frac{2}{3}\%$ of 54.
31. 75% of 36.
32. $83\frac{1}{3}\%$ of 30.
33. 80% of 200.
34. $6\frac{1}{4}\%$ of 64.
35. $37\frac{1}{2}\%$ of 72.
36. $83\frac{1}{3}\%$ of 42.
37. $87\frac{1}{2}\%$ of 96.
38. $16\frac{2}{3}\%$ of 75.
39. $12\frac{1}{2}\%$ of 60.
40. 10% of 2.07.
41. $33\frac{1}{3}\%$ of .6.

36. To find any per cent of a number. Since 75% of a number means .75 of it, 75% of 360 may be found by multiplying 360 by .75.

EXAMPLE. Find 5% of 480.

SOLUTION. 5% of $480 = .05 \times 480 = 24.$

$$\begin{array}{r} 480 \\ \underline{.05} \\ 24.00 \end{array}$$

Exercise 35

Find the value of

1. 5% of 320.
2. 7% of 250.
3. 8% of 4800.
4. 12% of 250.
5. 6% of 350 ft.
6. 65% of 980 yd.
7. 120% of \$400.
8. 17% of 3456.
9. 32% of \$38.50.
10. 97% of \$840.
11. $1\frac{1}{2}\%$ of 560.
12. $6\frac{1}{3}\%$ of 87.5.
13. $35\frac{1}{4}\%$ of 872.45.
14. $\frac{1}{2}\%$ of 96.
15. $.02\frac{1}{2}\%$ of 56,896.
16. 675% of 72.
17. 25% of \$484, in two ways.
18. $12\frac{1}{2}\%$ of 960.8.
19. $33\frac{1}{3}\%$ of 1000.
20. $37\frac{1}{2}\%$ of 54.09.
21. $62\frac{1}{2}\%$ of 8.4.
22. 60% of 395.07.
23. 10% of 50.004.

The number of which a per cent is taken is called the **base.** The per cent taken is called the **rate** or the **rate per cent.** The result of taking the per cent of the base is called the **percentage.**

Exercise 36

1. Representing the base by b, the rate by r, and the percentage by p, make a formula for finding the percentage when the base and rate are given.

2. Use the formula $p=rb$ to find p when,

a. $r=8\%$ and $b=20$.

HINT. Express r as a decimal. Here $r=.08$.

b. $r=25\%$ and $b=144$.
c. $r=43\%$ and $b=2.5$.
d. $r=2.5\%$ and $b=\$14,720$.
e. $r=16\frac{2}{3}\%$ and $b=7243$ acres.
f. $r=37\frac{1}{2}\%$ and $b=\$146,738$.

3. A family spends 20% of its income for rent, 30% for food, 25% for clothing, 10% for incidentals, and saves the balance. The family income is $1200 a year. How much does it pay for rent? How much does it save?

4. If a family saves $8\frac{1}{3}\%$ of its yearly income of $1575, how much does it save in a year?

5. The per cents of waste in the different cuts of beef are given in the following table:

Brisket	23.3%	Neck	31.2%
Rump	19.0%	Ribs	20.1%
Flank	5.5%	Round	8.5%
Chuck rib	53.8%	Shin	38.3%
Porterhouse	12.7%		

How much waste is there in a 12-pound cut of each of these kinds of beef? What is the average per cent of waste in a 12-pound cut?

6. In a certain room $33\frac{1}{3}\%$ of the pupils made a grade of exactly 75% on a test. There were 42 pupils in the room and there were 12 questions on the test. How many correct answers were given by all the pupils making 75% on the test?

7. How much water in 5 bushels of potatoes weighing 60 pounds to the bushel, if 78% of their weight is water?

8. A chicken is 44% water. How much water do we buy when we buy a five-pound chicken?

9. A student in a summer sold \$1500 of aluminum ware. For his pay he received 40% of the selling price of the aluminum sold. How much was he paid?

10. A certain kind of tire for automobiles formerly sold for \$24. The price has been advanced $12\frac{1}{2}\%$. What is the price now?

11. A bushel of corn plants eight acres. In an eighty-acre field 15% of the corn did not germinate. How many bushels failed to germinate?

12. A workman receives \$3.50 a day. His wages are increased 10%. How much does that increase his income in a year of 300 working days?

13. If a grocer buys berries at 40¢ a gallon and sells them at an advance of 100% how much does he make on 34 gallons?

14. A pupil has a garden 25 ft.×42 ft. He plants 8% of his ground in lettuce, 9% in radishes, $16\frac{2}{3}\%$ in beans, $33\frac{1}{3}\%$ in tomatoes, and the remainder in potatoes. The lettuce and radishes yield \$1 for each 7 square feet, the beans \$1 for each 25 square feet, the potatoes \$1 for each 30 square feet, and the tomatoes \$1 for each 35 square feet. He has spent for fertilizer and labor 8% of the returns from his crops. *How much does* he clear for his own work?

37. A per cent more or less than a number. When we speak of $\frac{1}{2}$ more than a number we mean the result of adding $\frac{1}{2}$ of that number to the number. One-half more than 10 is $\frac{1}{2}$ of $10+10$, or 15.

If Mr. Smith raised $33\frac{1}{3}\%$ more apples than Mr. Brown, who raised 360 bu., then Mr. Smith raised $33\frac{1}{3}\%$ of 360 bu. more than 360 bu., or 480 bu.

One-fourth less than a certain number means the result of subtracting $\frac{1}{4}$ of the number from the number. Twenty-five per cent less than 80 means $80-25\%$ of 80, which is 60.

Exercise 37

1. Find $\frac{1}{2}$ more than 60 ; $\frac{2}{3}$ less than 45.

2. Find $33\frac{1}{3}\%$ less than 384 ; 7% more than \$256.

3. The population of Indianapolis in 1910 was 233,650. What was the population after it had increased 5%?

4. Fred and Robert each get a salary of \$85 a month. Fred's salary is first raised 10% and then lowered 10%. Robert's salary is first lowered 10% and then raised 10%. Which now gets the better salary? Estimate the answer and then compute it.

5. A merchant found that his sales for a certain month amounted to \$8750. The following month they increased 4%, the next they increased 7%, the next they decreased 3%, the next they increased 1%, the next they decreased 4%. What was the amount of the sales for the last month? What were the average monthly sales for the six months?

6. The cost of a certain article was $8\frac{1}{3}\%$ less than the selling price, which was \$150. What was the cost?

7. A boy put \$50 in the savings bank with the understanding that the bank would increase it by 2% every six months. How much did he have in the bank at the end of the *first six months*? At the end of $2\frac{1}{2}$ years?

8. John finds that in his sixteenth year his height increased $6\frac{1}{4}\%$. At the beginning of the year his height was $5\frac{1}{2}$ feet. What was it at the end of the year?

9. The taxes in a certain city increased .3% over last year's taxes. If a man paid \$245 last year, what were his taxes this year?

10. A stalk of corn 20 in. high increased in height 45% in two weeks. The height of the corn after the increase was what per cent of its height before the increase?

11. A quantity of salt is dried until it loses 10% in weight. Its weight after the drying is what per cent of its weight before?

12. A hog loses 19% of its weight in being killed and dressed. If a hog weighs 280 pounds on foot, how much does it weigh dressed?

13. If new corn loses 3% of its weight in drying, is it better to pay 75 cents a bushel for new corn or 80 cents for old corn? A bushel of corn weighs 56 lb.

14. A city lot worth \$475 five years ago has increased in value 12% in the five years. How much is it worth now?

15. Make a formula for finding the weight, W, of an object which has increased in weight at the rate, r, from the weight, w.

16. Make a formula for finding the value, V, of an article which has decreased in value the per cent, r, from the value, v.

17. Can you give the answers to the first two exercises in Exercise 34 in the time stated there?

18. Give orally $12\frac{1}{2}\%$ of each of the following: 80, 120, 1000, 6.4, 20, .56.

19. Give orally $33\frac{1}{3}\%$ of each of the following: 18, .018, 200, \$5.70, $3\frac{1}{2}$ in.

38. To reduce any common fraction to per cent.

To change any common fraction to a decimal you have learned to divide the numerator by the denominator.

EXAMPLE 1. Reduce $\frac{3}{8}$ to a decimal.

SOLUTION. $\begin{array}{r} .375 \\ 8\overline{)3.000} \end{array}$ Therefore $\frac{3}{8} = .375$.

EXAMPLE 2. Reduce $\frac{3}{7}$ to per cent.

SOLUTION. $\begin{array}{r} .42\frac{6}{7} \\ 7\overline{)3.00} \end{array}$ Therefore $\frac{3}{7} = .42\frac{6}{7} = 42\frac{6}{7}\%$.

Exercise 38

Reduce the following :

1. $\frac{3}{4}$ to tenths ; to hundredths ; to thousandths.
2. $\frac{1}{3}$ to tenths ; to hundredths ; to thousandths.
3. $\frac{9}{10}$ to per cent.
4. $\frac{7}{8}$ to per cent.
5. $\frac{3}{11}$ to thousandths.
6. 3 to tenths ; to thousandths.
7. $2\frac{3}{4}$ to hundredths ; to tenths ; to thousandths.
8. $1\frac{5}{6}$ to per cent.
9. .3 to per cent.
10. $.2\frac{1}{2}$ to thousandths ; to hundredths ; to per cent.
11. $.0\frac{4}{5}$ to per cent ; to thousandths.
12. $.00\frac{2}{3}$ to per cent.
13. $\frac{5}{7}$, $\frac{8}{9}$, $12\frac{1}{2}$, $3\frac{1}{3}$, $.00\frac{1}{2}$ to per cents.
14. Make a rule for changing a common fraction to a per cent.
15. Repeat the rule for changing a per cent to a decimal.
16. Make a rule for changing a decimal to a per cent.
17. Change to per cents: .75, .18, .018, .003, 400, 2000.
18. What *per cent* of anything is $\frac{1}{6}$ of it? $\frac{2}{6}$? $\frac{3}{6}$? $\frac{4}{5}$? $\frac{1}{8}$? $\frac{3}{8}$? $\frac{5}{8}$? $\frac{7}{8}$?

39. To find what per cent one number is of another.

1. If your school had a team in a baseball league with 7 other teams, could you tell which team has the best standing after several games have been played? Do you know how a big league computes the percentages of its teams?

2. Suppose your team has played 20 games and has won 11 of them, what part of the games played has it won? What per cent of them has it won?

3. Team B has played 25 games and has won 14 of them. What part of its games has it won? What per cent of them?

4. Team C has played 30 games and has won 16 of them. What per cent of its games has it won?

5. Team D has played 27 games and has won 15. What per cent has it won?

6. Team E has played 28 games and has won 14 of them. What per cent has it won?

7. Which team has won the largest per cent of the games played? Which is second?

8. Beginning with the best team, name them in the order of their standing.

Notice that the standing of a team is a per cent of the number of games played. The number of games played is therefore the base.

In the above problems you have been required to find what per cent the number of games won is of the number of games played.

In finding what per cent one number is of another the second number, of which the per cent is taken, is the base. The first number, which is a certain per cent of the second, is the percentage, and it is required to find the rate per cent.

The rate per cent is found by dividing the percentage by the base, expressing the quotient as hundredths, then as per cent.

EXAMPLE. 12 is what per cent of 20?

SOLUTION. $12 = \frac{12}{20}$ of 20.

$\frac{12}{20} = .60 = 60\%$.

Therefore, $12 = 60\%$ of 20.

Exercise 39

1. 3 is what part of 5? What per cent of 5?
2. 18 is what per cent of 72?
3. What per cent of 96 is 30?
4. $2\frac{1}{2}$ is what per cent of 15?
5. 125 is what per cent of 625? Of 125? Of 25?
6. There are 15 boys in a room of 35 pupils. What per cent of the pupils are girls?
7. .3 is what per cent of 30? Of .03?
8. 2 is what per cent of 200?
9. 200 is what per cent of 2?
10. What per cent of 50 is 1000?
11. What per cent of 1000 is 50?
12. 1000 is what per cent of 1?
13. 1 is what per cent of 1000?
14. What per cent of 250 is 30?
15. 19.2 is what per cent of 600?
16. 85 is what per cent of 412, correct to .1%?
17. What per cent of 732 is 935, correct to .1%?
18. What per cent of 5432 is 1763, correct to .01%?
19. In a certain test 10 questions were given. John answered 8 correctly. What per cent did he make on the examination?
20. In the same examination Mary gave $8\frac{1}{2}$ correct answers. What per cent did she make?

21. Compute the per cent of games won by each team in this league :

Team	Games Won	Games Lost	Per Cent
1	64	24	
2	30	57	
3	66	20	
4	18	70	
5	48	31	
6	38	42	
7	27	58	
8	45	32	

22. In a rifle match each contestant fired 25 shots. A made 18 hits, B made 22, C 24, and D 21. What per cent of each man's shots hit the target?

23. A superintendent of schools wanted to know which room had the best attendance on a certain circus day. Room A had 3 pupils absent out of 40 ; room B had 7 absent out of 52 ; room C had 4 absent out of 32 ; room D, 5 out of 42 ; room E, 3 out of 36 ; room F, 6 out of 45 ; room G, 2 out of 25 ; room H, 4 out of 45 ; room I, 8 out of 40. What per cent of the pupils of each room were present? Name the rooms in the order of their attendance rank, the one having the best attendance coming first.

24. In 100 pounds of garden soil there are 14.41 pounds of water. What per cent of the wet soil is water? After the water has been taken out, its weight is what per cent of that of the dry soil?

25. An acre of soil 7 inches deep weighs about 2,000,000 pounds. If it contains 328.8 barrels of water, what per cent of the weight of the soil is water? Assume that a barrel *contains 4 cubic feet.* A cubic foot of water weighs $62\frac{1}{2}$ *pounds.*

40. To find a number of which a certain per cent is known.

1. The product of two numbers is 15. One of the numbers is 3. What is the other?

2. The product of two numbers is 85. One of the numbers is 5. Find the other.

3. One of the two factors of 117 is 13. Find the other factor.

4. What number multiplied by 2 gives 5 as a product?

5. What number multiplied by 5 gives 2 as a product?

6. If the product of two numbers is $22\frac{1}{2}$ and one of the numbers is $7\frac{1}{2}$, what is the other number?

7. The product of two numbers is 36. One of the numbers is .03. What is the other?

8. What number multiplied by .25 will give 284.3 as a product?

9. What number multiplied by 5% will give 24 as a product?

10. If the product of two numbers is given and one of the numbers is known, how may the other be found?

11. The formula $p=r\times b$ tells that the percentage is the product of the base and the rate per cent.

If the percentage and the base are known, how may the rate be found?

12. If the percentage and the rate are known, how may the base be found?

These answers give us these two rules:

The base equals the percentage divided by the rate per cent.

The rate per cent equals the percentage divided by the base.

We now have the three formulas:

(1) $p=b\times r$, (2) $b=\frac{p}{r}$, and (3) $r=\frac{p}{b}$.

The pupil should notice that in all computations with r r *is expressed* as a common or a decimal fraction.

Exercise 40

Use the formulas in solving the following exercises :

1. $r=32\%$, $p=4$. Find b.

SOLUTION. Here b is desired, so formula (2) must be used.

$$b=\frac{p}{r}=\frac{4}{.32}=12\tfrac{1}{2}.$$

2. $p=756$, $r=7\%$, $b=?$
3. $b=9.25$, $r=28\%$, $p=?$
4. $p=18.75$, $r=30\%$, $b=?$
5. $b=\$4806$, $p=\$81.20$, $r=?$
6. $b=1.005$, $r=33\tfrac{1}{3}\%$, $p=?$
7. $p=.096$, $r=12\tfrac{1}{2}\%$, $b=?$
8. $b=.85$, $p=.34$, $r=?$
9. Two inches is what per cent of 6 inches?
10. Six inches is what per cent of 2 inches?
11. A foot is what per cent of a yard?
12. An inch is what per cent of a foot?
13. A quart is what per cent of a gallon?
14. A pound Troy is 5760 grains and a pound avoirdupois is 7000 grains. A pound Troy is what per cent of a pound avoirdupois?
15. A centimeter is what per cent of an inch?
16. An inch is what per cent of a centimeter?
17. A liter is 1.0567 liquid quarts. A liter is what per cent of a liquid quart?
18. A square meter is 1.196 sq. yd. It is what per cent of a square yard? A square yard is what per cent of a square meter?
19. A square mile is 2.59 square kilometers. It is what per cent of a square kilometer? A square kilometer is what per cent of a square mile?

20. On Monday William adds a certain example in 20 seconds, and John adds it in 22 seconds. On Tuesday each gets it in 21 seconds. By what per cent has William's time increased? By what per cent has John's decreased?

21. A certain mechanic receives \$75 a month. On account of a panic his wages are reduced 10%. By what per cent must his new wages be increased to give him his first wages?

22. A man paid \$135 for a horse and sold him so that he gained 10% of the purchase price. How much did he gain?

23. A poultryman set 432 eggs and 360 hatched. What per cent of the eggs set hatched?

Exercise 41. Problems of the newsboy

1. A newsboy has saved \$75 from his earnings. If he puts it in a savings bank and leaves it there half a year, the bank will pay him 2% of the amount for the use of it. Another newsboy offers to pay him \$1.75 for the use of the money for the six months. Which of these plans should he follow?

2. He lends the money to the newsboy for the six months, and puts the amount he receives from the newsboy in the savings bank for the remainder of the year. How much does the savings bank owe him at the end of the year?

3. By selling papers during this year he earns \$18 a month and his expenses are \$16.50 a month. How much of his earnings does he save this year? What are his total savings at the end of the year?

4. At the beginning of the second year he puts all his savings in the savings bank, which pays him 4% for its use for a year. He is now offered the job of selling popcorn in a moving picture theater. He is to receive 35% of all the *money he takes in.* He feels sure that he can sell each day

at least 50 bags at 5 cents each. Should he quit selling newspapers and accept the new job?

5. He accepted the new job. The first week his daily sales were as follows : 75 bags, 45 bags, 86 bags, 94 bags, 36 bags, 107 bags. How much did he receive each day? How many bags did he sell in the week? Find in two ways his pay for the week.

6. His average daily sales for the year were 62 bags a day. He worked 295 days that year. What were his total sales? What was his pay for the year?

7. When he sold papers and earned \$18 a month his expenses were \$16.50 a month. He found they had increased 18% the year he sold popcorn. How much were his expenses this second year? How much did he save from the sale of popcorn the second year?

8. How much did the savings bank owe him at the end of the second year? What were his total savings at the end of the second year?

9. The third year the manager of the theater offers to sell him the right to sell both popcorn and peanuts for \$150 for the year, or for 20% of his sales, as he chooses. He estimates that he can continue to sell daily at least 35 bags of popcorn and 30 bags of peanuts at 5 cents each. He counts upon working at least 285 days. Which of the two offers of the manager should he accept?

10. Instead of buying his corn in bags at \$2.25 a hundred, he finds that he can buy a cornpopping machine for \$395. The raw corn will cost him 0.6¢ for each bag of popped corn. The butter, salt, bags, and gas will cost 1.2¢ a bag. To get the money to pay for the machine and the cost of the popped corn he must pay 7% of the amount for its use for one year. Will it pay him to borrow the money and buy the machine?

41. Problems of food values.

The food materials needed by the body to build up its tissues and to supply it with energy are protein, carbohydrate, fat, mineral matter, and water.

In studying the value of any food it is necessary to know how much building material and how much energy it produces.

The energy of the body is measured in terms of heat. Heat is measured in **calories**. A calorie is the amount of heat necessary to raise the temperature of a pound of water about 4 degrees Fahrenheit.

Exercise 42. Food values

1. It has been found that

One pound of fat produces . . . . 4082 calories of heat.
One pound of protein produces . . 1814 calories of heat.
One pound of carbohydrate produces 1814 calories of heat.
A pound of fat produces how many times as much heat as a pound of protein or of carbohydrate?

2. How many calories of heat are produced by an ounce of each of these substances?

3. How many calories of heat are furnished by a pound of butter which is 85% fat and 1% protein?

4. How many calories are furnished by a pound of sirloin steak which contains 16.5% protein and 16.1% fat?

5. A quart of milk contains the following elements:

Element	Water	Fat	Protein	Carbohydrate	Mineral Matter	Total
Ounces	29.93	1.38	1.13	1.72	2.4	36.56

Find the per cent of each of the elements in the quart of milk. Answer correct to .1%.

6. Compute the number of calories furnished by a quart of milk. Only fat, protein, and carbohydrate furnish energy.

7. The following table gives the per cent of the elements found in certain foods:

Article of Food	Waste	Water	Protein	Fat	Carbohydrate	Mineral Matter
Sirloin steak (as purchased)	12.5	54.0	16.5	16.1		.9
Eggs (as purchased)	11.1	65.6	11.9	10.5		.9
Milk		87.0	3.3	4.0	5.0	.7
White bread . . .		35.3	9.2	1.3	53.1	1.1
Potatoes	20.0	62.6	1.8	.1	14.7	.8

Find the number of calories furnished by 1 pound of each of the above foods.

8. How many pounds of water in a bushel of potatoes? How many pounds of waste? A bushel of potatoes weighs 60 pounds.

9. If the price of a food were to depend only upon its value as an energy producer, that is, upon the number of calories of heat that it produces, what should be the price of eggs per dozen when sirloin steak costs 25 cents a pound? Assume that a dozen eggs weigh 1.5 lb.

10. Find also what should be the cost of a loaf of bread weighing 12 ounces when potatoes cost $1.50 a bushel.

11. Find how many calories of each of the above foods can be bought for 10 cents if sirloin steak costs 25 cents a pound; eggs, 30 cents a dozen; milk, 10 cents a quart; bread, 10 cents for a loaf weighing 24 ounces; and potatoes, $1.50 a bushel. Use the weights of milk, eggs, and potatoes given in previous problems.

12. Of the above foods which has the highest and which has the lowest per cent of waste and water?

CHAPTER V

APPLICATIONS OF PERCENTAGE

Profit and Loss

42. The per cent of gain or loss. It is sometimes necessary to decide which of two investments is the more profitable. To do this it is usual to compute the per cent which is gained on each dollar invested.

Example. A boy paid $5 for some hens which he sold for $6. He also paid $8 for some pigeons which he sold for $9.20. He still has $12 to invest. Is it better to invest it in hens or in pigeons?

Solution. On the $5 invested in hens he gained $1.

Therefore the amount made on the hens is 20% of the amount invested.

On the $8 invested in pigeons he gained $1.20.

The gain on the pigeons is, then, 15% of the sum invested.

If he should invest his $12 in hens, he may expect to gain 20% of $12, which is $2.40.

If he invests in pigeons, he may expect to make 15% of $12, which is $1.80.

Therefore it is better to invest in hens.

In computing profit or loss it is the usual custom to use the cost as the base.

Exercise 43

1. An article was bought for $4 and sold for $6. How much was the gain? The gain was what part of the cost? The gain was what per cent of the cost?

2. Find the per cent of gain on the following articles :

Cost	$12	$3.50	$0.40	$125	$0.01	$75
Selling price . . .	15	7.00	0.80	150	0.03	90

3. Find the selling price of an article which cost \$8 and was sold at a gain of 25%.

4. Find the selling price in each of the following cases :

Cost	\$1.50	\$.06	\$2000	\$0.01	\$475.
Per cent of gain . .	10%	25%	7%	300%	20%.

5. Tell the amount of profit or loss and the rate of profit or loss, if an article is bought for

a. \$1 and sold for \$2.
b. \$2 and sold for \$1.
c. \$1 and sold for \$3.
d. \$3 and sold for \$1.
e. \$1 and sold for \$10.
f. \$10 and sold for \$8.
g. \$1.10 and sold for \$1.
h. \$1.20 and sold for \$1.
i. \$.01 and sold for \$1.
j. \$4 and sold for \$4.

6. Tell the amount of profit or loss and the rate of profit or loss if an article that sells for a dollar was bought for 50¢ ; for 25¢ ; for 10¢ ; for 1¢.

7. A cow costing \$125 gave 6723 pounds of milk in one year. The cost of her feed and care for the year was \$154. The milk was sold for 10 cents a quart. Allowing 8.6 pounds of milk to the gallon, what per cent did the owner make on the cost of the cow and the expense of keeping her?

8. An investment of \$78 in fertilizer for a certain field increased the value of the crop taken from it by \$117. What was the per cent of gain on the amount spent for fertilizer?

9. A furniture dealer buys 6 couches at \$35 each and 8 sets of dining chairs at \$33 a set. He sells the couches at a profit of 40% and the dining chairs at a profit of 35%. For how much did he sell this lot of furniture?

10. A hardware dealer gets in a lot of scissors costing 62¢ a pair, of knives costing 38¢ apiece, of knives costing 78¢ apiece, of skates costing 90¢ a pair, and of razors costing \$2.75 apiece. He marks them, to the nearest five cents, so as to make a profit of 30%. Find the marked prices.

43. To find the cost when the selling price and the gain or loss per cent are known.

1. What sum is 25% more than \$4?

2. What sum is 25% less than \$4?

3. 10% more than 50 is what per cent of 50?

4. 8% less than 200 is what per cent of 200?

5. 30% more than any number is what per cent of the number?

6. 2% less than any number is what per cent of that number?

7. An article bought for \$8 is sold at a gain of 25%. What is the selling price? The selling price is what per cent more than the cost? The selling price is what per cent of the cost?

8. An article is sold at a gain of 6%. The selling price is what per cent of the cost?

9. A horse is sold at a loss of 15%. The selling price is what per cent of the cost?

10. 15% of a number is 75. What is the number?

11. 130% of a number is 285. What is the number?

12. By selling an article for \$5 a boy made a profit of 25% on it. The selling price is what per cent of the cost? What was the cost?

EXAMPLE. A hardware dealer marks a line of goods to sell at a profit of 40%. He finds a bucket marked to sell at \$1.05, but the cost is not marked. Find the cost for him.

SOLUTION. Selling price = 40% more than the cost.
Selling price = 140% of the cost.
Cost price = \$1.05 ÷ 1.40
= \$.75.

$$\begin{array}{r} .75 \\ 1.40\overline{)1.05} \\ \underline{98} \\ 70 \\ \underline{70} \end{array}$$

Exercise 44

1. John sold his dog for $[illegible] at a loss of 25%. What had he paid for the dog?

2. Find the cost of the following articles:

Selling price	$44	$20.70	$3.71	$.43.
Gain per cent	40%	15%	6%	35%.

3. A furniture dealer has his stock marked to sell at a gain of 25%. Find the cost of a davenport marked at $75, a mirror at $3.75, a set of chairs at $25, and a rug at $45.

4. A grocer can sell strawberries at $3.60 a crate. How much should he pay for them to make 20% on them?

5. Mr. Jones finds that his increased yield from the use of a ton of fertilizer is 47 bushels of corn which he sells at 85 cents a bushel. He has agreed to pay for the fertilizer a price which will leave him a profit of 10% of its cost. What price should he pay the dealer for the fertilizer?

6. A real estate dealer can sell a certain lot for $575. He wants to buy it at a price so that he can make 15% on his investment. What should he pay for it?

7. A farmer claims that by a certain method of cultivation he has increased the yield of a field 15%. His yield with this cultivation was 31 bushels an acre. What would it have been without this method of cultivation, according to his estimate? Find result to the nearest bushel.

8. A man sold a farm for $5906.25. This was a gain of 25% on his total investment in the farm. Since buying the farm he had spent on improvements 5% of the purchase price of the farm. What was the original purchase price?

[illegible] Martin sold a flock of 144 sheep for $1925. Since [illegible] them their value had increased 10% and the number [illegible] had increased 20%. How much had he paid [illegible] the original flock?

44. Profit and loss formulas.

1. If you are told the cost of an article and the gain at which it is sold, how can you find the selling price?

2. Representing the cost by c, the gain by g, and the selling price by s, make a formula for finding the selling price.

3. If you know the cost of an article and the rate of gain, how can you find the gain?

4. Representing the rate of gain by r, make a formula for finding the gain when the cost and the rate of gain are known.

5. Representing the loss by l and the rate of loss by r, make a formula for finding the loss when the cost and the rate of loss are known.

6. If you know the cost and the rate of gain, how can you find the selling price?

7. Make a formula for finding the selling price when the cost and the rate of gain are known.

8. Make a formula for finding the selling price when the cost and the loss are known.

9. Make a formula for finding the selling price when the cost and rate of loss are known.

The answers to the preceding questions give the following formulas:

1. $s=c+g$.	4. $s=c+rc$.
2. $g=rc$.	5. $s=c-rc$.
3. $l=rc$.	

Exercise 45

1. Make a problem which tells you that $c=\$125$ and $g=\$25$, and requires you to find s. Find s, using formula 1.

2. Make a problem which tells you that $c=6¢$ and $r=33\frac{1}{3}\%$, and requires you to find l. Find l.

3. Make a problem which tells that r (the rate of gain) $=8\frac{1}{3}\%$ and that $c=\$69.45$, and requires you to find s. Find s.

4. Make and solve a problem requiring you to find g and telling you that $r = 18\%$ and $c = \$345$.

5. Replace the question marks in this table with values for the letters.

	c	s	r	r'	g	l
a.	\$275	?	?		\$25	
b.	\$.20	?	45%		?	
c.	?	\$270		?		\$30
d.	\$6.25	?		20%		?
e.	\$9265	\$8400		?		?
f.	\$1.25	\$3.25	?		?	
g.	?	\$42	$12\frac{1}{2}\%$		?	
h.	?	\$370		30%		?

6. In a certain city the gas company reduces the consumer's bill 10% if it is paid within ten days after it is sent out. Mr. Allison's bill was \$4.80. He paid it within the ten days. How much did he pay? What per cent of the amount paid did he save by prompt payment?

7. Mr. White bought 18 tons of coal in October at \$5.40 a ton, when he might have bought it in August at \$4.25 a ton. What per cent (of the cost of the coal) did he lose by buying in October?

8. By buying coal in August Mr. Black pays only \$4.25 a ton for 18 tons. If he had waited until October he must have paid \$5.40 a ton. What per cent did he save by buying in August?

9. In a certain year an experiment station reported the cost of raising an acre of corn as follows:

Rent	\$5.203	Cultivating	\$1.415
Plowing	1.276	Harvesting	1.688
Harrowing	.305	Interest and depreciation on machinery . .	.380
Disking	.481	Miscellaneous	.578
Seed	.283		
Planting	.408		

The acre produced 38.4 bushels. At what price per bushel *must it be sold* to yield a profit of 10%?

Commercial Discount

45. Meaning of commercial discount. Retail merchants buy their stocks either directly from the factory or from a wholesale dealer or jobber. In the catalog which he sends to the merchant the wholesale dealer quotes the prices of the different articles. It is customary to quote on a separate sheet the discounts which may be deducted from the catalog or list price. The price of a certain stove may be \$63 in the catalog but the discount sheet may state that a discount of 40% is allowed. Often a second discount of 2% or 3% is allowed for paying cash, and occasionally a third discount is allowed.

Frequently a retail merchant allows a discount from the regular price of an article, and sometimes he allows a second discount for the payment of cash.

Example. The catalog or list price of a davenport is \$90 but the discount sheet shows discounts of 40% and 5%. What price must the merchant pay for the davenport?

Solution. 40% of \$90 = \$36, the first discount.
Deducting this discount the price then is \$54.
5% of \$54 = \$2.70, the second discount.
\$54 − \$2.70 = \$51.30, the price which the merchant must pay.

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Instead of taking 40% of the list price from the list price, the same result might have been obtained by finding 60% of the list price. And instead of taking 5% of the 60% of the list price and subtracting it we might have taken 95% of the 60% of the list price. But 95% of 60% of the list price is the same as 57% of the list price, and 57% of \$90 is \$51.30. This suggests this rule for finding the cost of an article on which two or more successive discounts have been allowed:

Subtract each of the discount per cents from 100%. Find the product of the resulting per cents and multiply the list price by the result.

Exercise 46

1. A book dealer receives a bill amounting to $450 for books. The bill states that a discount of 2% is allowed if the bill is paid within thirty days. The next day he pays the bill. How much does he send?

2. Mary sells her chickens for which she has been asking $15, at a discount of $33\frac{1}{3}$%. How much does she get for them?

3. The list price of a certain book is $1.60. What will it cost a dealer who is allowed a discount of 20%?

4. At a closing out sale a discount of 25% was allowed on every article in a furniture store. What was the selling price of a chair previously marked $15? Of a dining table marked $22? Of a set of dining chairs marked $27.50? Of a bed marked $82?

5. The furniture dealer of the preceding problem had bought the furniture at discounts of 40% and 5% off the following list prices: The chair, $18; the dining table, $25; the set of dining chairs, $32; the bed, $94.50. How much did he pay for each?

6. A wholesale house allows discounts of 20% and 10%. On a bill of goods amounting to $1200, an inexperienced clerk allowed a single discount of 30%, thinking the result would be the same. How much did his mistake cost his employer? Why is a single discount of 30% greater than two successive discounts of 20% and 10%?

7. A retail merchant wishes to buy 40 yards of silk. He asks two wholesale houses to quote prices. The first quotes $2.25 a yard, less discounts of 25% and 5%. The second quotes $2.25 a yard, less discounts of 20% and 10% and an additional 1% for cash. Which is the better bargain? How much better is it?

8. A hardware dealer finds a plow marked to sell for $35. He knows this stock was marked to sell at an advance of 40%. How much had he paid for this plow? He sells it at a discount of 20% of the marked price. How much does he make on his investment?

9. On a bill of goods for $756 which is better, discounts of 25% and 10%, or a single discount of 33%?

10. On a bill for $60, which is best, discounts of 35% and 5%, or 5% and 35%, or a single discount of 39%?

46. Bills. When a bill of goods is ordered or services rendered for which payment is expected a **statement of account** or **bill** is sent to the purchaser by the one from whom they are purchased. Below is a common form of such a bill.

A. M. SKELETON & CO.

PUBLISHERS

Chicago, Illinois

December 14, 1924

SOLD TO John C. Black,
Marysville, Kentucky

TERMS: 2% 30 days.

		25 Robinson Elementary Algebra 1.25 20%	25	00		

The $1.25 is the list price of each book, the 20% is the first rate of discount, and the $25.00 is the amount the buyer must remit if he does not choose to pay within the 30 days. If he does choose to pay within 30 days he may also deduct the additional discount of 2%.

Exercise 47

1. Find the amount of this bill if paid within ten days.

A. R. SMITH & CO.

Chicago, Illinois
November 8, 1918

SOLD TO Ralph C. Hammond,
Madison, Wisconsin

TERMS: 1% 10 days.

15	Kipling Jungle Book	1.50	10%	
4	Carver Essays Social Justice	2.00	20%	
1	O. Henry Complete	24.00	25%	
8	Wells Mr. Britling	1.50	15%	
45	Jones Complete Arithmetic	.60	4%	
5	Riis Making of an American	2.00	25%	
7	Duncan Billy Topsails	1.60	15%	
3	Hugo Les Miserables	2.25	5%	

2. Write the bill to be sent to J. W. Comstock, Canton, Ohio, who ordered the following goods from John A. Driscoll, Columbus, Ohio. A discount of 6% is allowed if payment is made within 10 days, of 5% if made within 30 days, and of 4% if made within 60 days. Compute the amount required to pay the bill under each condition.

15 yd. ribbon at \$.21½, 17 yd. ribbon at \$.27½, 5 yd. ribbon at \$.37½, 6 yd. ribbon at \$.53, 8 petticoats at \$3.25, and ¼ thousand crewel needles at \$1.50 per thousand.

3. E. E. Wilson ordered the following bill from the Loxa *Tire Company*: 1 Loxa tire, 34″×4″, \$32.50; 1 Loxa inner tube, 34″×4″, \$4.50; 10 gallons cylinder oil at 65¢; 1 tire

cover, \$2.60; 1 tire pump, \$3.25; 2 Loxa Special spark plugs at 75¢. Write the bill and compute its amount allowing 5% discount for cash.

4. Mrs. Appleby sold to a customer the following produce: ½ bu. potatoes at 90¢ a bushel, 24 cabbages at 16¢, 82 lb. of chickens at 18¢, 35 dozen eggs at 47¢, and 56 lb. of butter at 52¢. Write the bill to be sent to the customer, R. J. Brown, and compute its amount, allowing him 3% for cash.

5. Copy the following bill. Compute the cost of each item and enter the cost in the column where it is checked. Note the discount allowed at the top of the different columns, and compute the amount of the bill which was paid at the end of 20 days.

LACEY & SON

Chicago, Illinois
November 13, 1918

SOLD TO H. W. Alexander,
Urbana, Illinois

		10 days less 2% 30 days less 1%	60 days less 4% 30 days less 5% 10 days less 6%
10 doz Handkerchiefs	2.15		✓
23 Ribbon 333	.32½		✓
1 doz. Gowns 5375	4.00	✓	
1 doz. " 5376	4.50	✓	
3 doz. " 5221x	4.50	✓	
12 gr. Buttons 21	.37½	✓	
2½ gr. " 45	4.00	✓	
1¼ gr. " 40	3.50	✓	
3 gr. " 16	.65	✓	

Exercise 48. Problems of the farmer

1. A farmer raised 42 bushels of corn per acre on 20 acres. The cost per acre of raising the corn was $16.25. He sold the corn for 82¢ a bushel. What per cent did he gain?

2. What per cent of the cost of the corn would the farmer of the preceding problem have made, if he had fed it to hogs, making a gain of 100 pounds for each 9 bushels of corn, and had sold them at $8.75 a hundred pounds?

3. A rectangular corn field is 80 rods long and 40 rods wide. It costs $10 an acre for rent, $1.50 an acre to plow the ground, $1.50 an acre to harrow it, $.40 an acre to plant it, $2 an acre to cultivate it, and $2.40 an acre to husk and haul the corn to the crib. One-half peck of corn at $1.60 a bushel was planted to the acre. What was the total expense of the field of corn? What was the expense per acre?

4. The yield per acre of the field of the preceding problem was 46 bushels. The corn was sold at 72¢ a bushel. What per cent of profit did the renter make?

5. In an experiment in feeding laying hens two pens of 25 hens each were given the same grain ration. Skim milk was added to the ration in the first pen. The average cost of feed per hen for a year in the first pen was $1.09 and in the other was $.715. The average number of eggs produced by the hens in the first pen was 137 and by the hens in the second pen was 37. The average price received for the eggs was 31.1¢ a dozen. What was the difference in the values of the eggs from the two pens? By what per cent was the value of the eggs produced increased by adding skim milk to the ration? Find the profit from each pen.

6. Seeds should generally be tested before planting. This *may be done* by planting a number of them in a seed germi-*nator and finding* what per cent of the number planted germi-

nate. If the vitality of the seed is said to be 95, it means that 95% of the seed planted germinate.

A farmer tests 100 grains of his seed corn and finds that only 65 germinate. What is its vitality? He refuses to plant this seed and buys from a neighbor seed testing 95. His yield of corn from this seed was $47\frac{1}{2}$ bushels per acre. What should his yield have been if his seed had had 100% vitality? What should it have been if he had planted his own corn with vitality 65?

7. Clover seed is sometimes of low vitality and also contains weed seed that much resembles clover seed. A farmer finds that his clover seed contains 4% dodder seed. The real clover seed has a vitality of 94. If his seed were pure and had vitality of 100 he would expect a crop of 2 tons per acre. How much may he expect from this seed?

8. When a bushel of Yellow Dent corn on the cob is shelled, the weight of the shelled corn is usually about 88% of the weight of the corn on the cob. The legal weight of a bushel of shelled corn is usually 56 pounds and of a bushel of ear corn 70 pounds. If a bushel of Yellow Dent corn in the ear weighs 70 pounds, how much will the shelled corn from this bushel weigh?

It costs about $1\frac{1}{4}$¢ a bushel to have corn shelled. A farmer has 1200 bushels of Yellow Dent corn in the ear. He can sell it for 85¢ a bushel. Is it better to sell it shelled or in the ear? How much better?

9. If a pasture feeds 2 cows to every 5 acres how much does a thirty-acre pasture earn during a five months' season at $2 a month per cow? How much can be paid for this land so that the buyer may receive an annual income of 6% on the purchase price allowing $1 an acre for expenses?

10. *Find the estimated average cost, yield, and profit per acre of the three principal crops in your county.*

47. Problems of the dairyman.

Milk as it comes from the cow is called **whole milk.**

By using a machine called a **separator** whole milk is separated into **cream** and **skim milk.**

A part of whole milk is pure fat, called **butter fat**, and almost all of this is taken out with the cream by the separator.

Butter is the butter fat plus salt, water, and some of the milk. The amount of butter which can be made from a given amount of milk is usually estimated as $1\frac{1}{6}$ times the butter fat it contains.

The expression "100 pounds of 4% milk" means that there are 4 pounds of butter fat in the 100 pounds of whole milk. The expression "20 pounds of 25% cream" means that 25% of the 20 pounds of cream is butter fat.

Whole milk is estimated as weighing 2.18 pounds to the quart, and cream as weighing 2 pounds to the quart.

Exercise 49

1. How much butter fat in 800 pounds of 3% milk? In 1275 pounds of $4\frac{1}{2}$% milk?

2. The milk of a certain Jersey cow contains 32 pounds of butter fat in 640 pounds of milk. What is the per cent of butter fat?

3. If all the butter fat in 200 pounds of $4\frac{1}{2}$% milk is taken out in the cream, how much 25% cream is made?

HINT. The amount of butter fat is 25% of the amount of cream.

4. How much butter can be made from 360 pounds of 5% milk? How much 20% cream?

HINT. See above the relation between butter and butter fat.

5. How much butter fat in 42 pounds of butter?

6. Twenty pounds of cream are taken from 100 pounds of 4% milk and the skim milk is then found to contain .02%

of butter fat. What per cent of the fat is lost in the skim milk?

7. Two neighbors have six dairy cows each. Record is kept of the amount and quality of the milk of each cow as follows:

	Herd No. 1		Herd No. 2	
Cow No.	Pounds of Milk per Year	Per Cent Butter Fat	Pounds of Milk per Year	Per Cent Butter Fat
1.	4204	4.07	12569	4.09
2.	3236	4.05	13164	3.70
3.	8944	2.99	9829	4.34
4.	6597	3.50	10380	4.41
5.	5548	3.85	16582	3.45
6.	4475	4.00	11146	4.20

a. Find the total amount of butter fat for each herd.

b. Find the per cent of butter fat for all the milk of herd No. 1.

c. Find the per cent of butter fat for all the milk of herd No. 2.

Find the value of each herd's product if sold as

d. Butter fat at 32¢ a pound.

e. Butter at 35¢ a pound.

f. Milk at 9¢ a quart.

g. Twenty per cent cream at 40¢ a quart.

h. Find the average annual value of the product per cow of each herd if sold as butter fat at 32¢ a pound.

i. The average cost of feed per cow in herd No. 1 is $62 and in herd No. 2 is $73. How much more than the cost of the feed does each man receive from his herd when the product is sold in each of the above forms? Which cows *are not paying* for their feed, if butter fat is sold as in h?

Exercise 50. Review

1. Write as per cents : .48, 2.5, 19, .005, $\frac{5}{6}$, $\frac{3}{8}$, $\frac{3}{4}$, 700, .020$\frac{1}{3}$.

2. Write as decimal fractions : 27%, .9%, 786%, .03%, $\frac{1}{4}$%, 1.02%, $\frac{5}{6}$%.

3. Give the per cent equivalent to each of these : $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{6}$, $\frac{2}{3}$, $\frac{5}{6}$, $\frac{1}{7}$, $\frac{1}{8}$, $\frac{7}{8}$, $\frac{5}{8}$, $\frac{3}{8}$, $\frac{1}{9}$, $\frac{3}{5}$, $\frac{2}{5}$, $\frac{1}{12}$. Practice until these results can be given in 30 seconds or less.

4. State the formula to be used (*a*) to find a per cent of a number ; (*b*) to find what per cent one number is of another; (*c*) to find a number when a certain per cent of it is given.

5. Find .0$\frac{1}{3}$% of 1.2.

6. $8.40 is $\frac{1}{8}$% of what number?

7. Find 96.84% of $18.72.

8. A yard is what per cent of a meter?

9. 80 is what per cent more than 75?

10. 75 is what per cent less than 80?

11. What number is 20% less than 6?

12. 6 is 20% less than what number?

13. A boy is 4 ft. 2 in. tall. If he increases 5% in height each year for 3 years, how tall will he then be?

14. The estimated wealth of the United States in 1917 was $250,000,000,000. The first Liberty Loan was for $2,000,000,000. The loan was what per cent of the estimated wealth?

15. Find out the present estimated wealth of the United States and the amount of indebtedness. The indebtedness is what per cent of the wealth?

16. A certain steel company earned $6,200,000 on an investment of $17,184,000. The earnings were what per cent of the investment?

17. A business man has invested \$5400 which yields 5% annually ; \$2000 which yields 4% ; \$1200 which yields 7% ; and \$800 which yields $8\frac{1}{2}$%. Find the rate of income on the total investment.

18. The value of the crops of the United States in a certain year was estimated to be \$21,000,000,000. This was an increase of \$8,500,000,000 over the value of the previous year. What was the per cent of increase?

19. Practice until you can write the sums on page 5, in the time indicated.

20. A square piece of cloth whose edge is 1 yd. shrinks until its edge is 35 in. By what per cent is the edge decreased? By what per cent is the area decreased?

21. One gram of fat produces 9 calories of heat ; protein and carbohydrate produce each 4 calories per gram. Find the number of calories produced by a kilogram of beefsteak, a kilogram of bread, and a liter of milk. See problem 7, page 82. A liter of milk weighs 1032 grams.

22. An agricultural experiment station reports the following data from a farm in Indiana concerning the profits from poultry in the two successive years :

	FIRST YEAR	SECOND YEAR
Average number of hens	331.2	325
Average number of dozen eggs per hen .	9.24	8.86
Average selling price per dozen	\$.173	\$.20
Total expenditures	\$282.74	\$297.61
Number of hours of labor	746	723

Find for each year, (*a*) the total income, (*b*) the average profit per hen, (*c*) the profit per hour on the labor, and (*d*) the rate per cent of increase of total profit the second year over *the first.*

CHAPTER VI

INTEREST

48. Meaning of interest. Money paid for the use of money is called **interest.**

The amount of interest to be paid depends upon the amount of money borrowed and the length of time it is kept. It is usual to charge as interest an agreed upon per cent of the money borrowed for each year that it is kept.

Henry wants to buy a plow to cost $50. He borrows the money from William Doyle and agrees to return the money at the end of one year and to pay 6% of the amount borrowed as interest for its use for the year. How much interest must he pay? How much money must he pay Mr. Doyle at the end of the year? If he borrows the money for two years, how much interest must he pay? If for 6 months?

The per cent to be paid each year is stated as a certain per cent per annum and this per cent is called the rate of interest. Per annum means "for a year."

Exercise 51

1. A man borrows $500 and agrees to pay 5% interest each year. If he keeps it one year, how much interest does he owe? If two years? Three years? Six months?

2. Answer each of the preceding questions if the rate of interest were 8% instead of 5%; if it were 7%; if it were 4%; 3%.

3. Find the interest on $150 for 5 years at 3%; at 6%; at 9%.

4. Find the interest on \$75 for 1 year at 8% ; for 6 months at 8%.

5. What is the interest on \$300 at 6% for 3 months?

6. Find the interest on \$4560 for 5 years at 4% ; for 6 years at 6% ; $2\frac{1}{2}$ years at 5%.

7. If \$375 is borrowed for 1 yr. 6 mo. at 8%, what sum must be paid at the end of that time?

Find the interest to the nearest cent on the following loans :

	Principal	Time	Rate
8.	\$100	1 yr.	4%
9.	\$300	2 yr.	5%
10.	\$250	4 yr.	2%
11.	\$500	$1\frac{1}{2}$ yr.	6%
12.	\$40	6 mo.	5%
13.	\$35	4 mo.	4%
14.	\$275	1 yr.	7%
15.	\$45.50	2 yr. 4 mo.	8%
16.	\$562	1 yr. 3 mo.	5%
17.	\$7800	4 yr.	$4\frac{1}{2}$%
18.	\$2890	2 yr. 6 mo.	6%
19.	\$5.75	6 mo.	8%
20.	\$5.75	3 mo.	8%
21.	\$5.75	3 mo.	4%
22	\$5.75	9 mo.	7%
23.	\$3870	3 mo.	8%
24.	\$3462.25	4 mo.	5%
25.	\$742.50	1 yr. 6 mo.	7%
26.	\$4200	8 mo.	4%
27.	\$678.25	2 yr.	5%
28.	\$58.50	1 yr. 6 mo.	2%
29.	\$275	4 yr.	6%
30.	\$850	3 mo.	7%

49. Expressing time as years. Since interest is computed at a certain rate for one year, it is necessary to be able to express any number of months and days as years.

Six months is what part of a year? What part of a year is 3 mo.? 4 mo.? 2 mo.? 5 mo.? 10 mo.? 7 mo.? 11 mo.?

In computing interest it is usual to consider 30 days a month and 12 months a year, and therefore 360 days a year.

What part of a month is 3 days? 10 days? 5 days? 6 days? 12 days? 18 days? 23 days?

What part of a month is 15 days? What part of a year is 1 month? $\frac{1}{2}$ of a month?

What part of a year is 15 days? 10 days? 3 months 15 days?

Since we consider 360 days a year, what part of a year is 1 day? 3 days? 17 days? 50 days?

To express a number of years, months, and days as years, first express the months and days as days, divide this number of days by 360, and add this fraction to the number of years given.

EXAMPLE. Express as years 2 yr. 3 mo. 20 da.

SOLUTION. 3 mo. 20 da. = 90 da. + 20 da. = 110 da.

110 da. = $\frac{110}{360}$ of a year = $\frac{11}{36}$ of a year.

2 yr. 3 mo. 20 da. = $2\frac{11}{36}$ yr.

Exercise 52

Express as years:

1. 6 months ; 3 months ; 7 months.
2. 30 days ; 15 days ; 10 days ; 3 days ; 12 days.
3. 2 mo. 15 da. ; 1 mo. 10 da. ; 6 mo. 9 da. ; 7 mo. 2 da.
4. 1 yr. 15 da. ; 2 yr. 5 mo. ; 1 yr. 2 mo. 15 da.
5. 3 yr. 15 da. ; 5 yr. 7 mo. ; 2 yr. 1 mo. 8 da.
6. *Find the interest* on $1250 for 1 yr. 5 mo. 10 da. at 6%.

50. The interest formula. How is the interest found on a um of money at a certain rate for one year? For a number f years?

If the sum of money is p dollars, the rate per annum r, and he number of years t, make a formula for finding the interest.

Remember that in this formula the time t must be expressed n years and that the rate r is the rate per annum.

The formula $i = prt$ should be remembered.

Example. Find the interest on \$2400 for 2 yr. 7 mo. 5 da. t 6%.

Solution.

2 yr. 7 mo. 5 da. $= \frac{935}{360}$ yr. $= t$.

$i = prt.$

$i = \$2400 \times \frac{6}{100} \times \frac{935}{360}$

$= \$374.$

2 yr. = 720 da.
7 mo. = 210 da.
5 da. = 5 da.
935 da.

$$\frac{\overset{\overset{2}{\cancel{24}}}{\cancel{2400}} \times \cancel{6} \times \overset{187}{\cancel{935}}}{\cancel{100} \times \underset{\underset{\cancel{5}}{\cancel{60}}}{\cancel{360}}} = 374.$$

Exercise 53

Use the formula in finding the interest on the given mounts following :

	Principal	Time	Rate
1.	\$4000	4 mo. 20 da.	4%
2.	\$5600	1 yr. 3 mo.	6%
3.	\$150	3 yr. 8 mo.	8%
4.	\$2500	5 mo. 15 da.	5%
5.	\$175	2 yr. 10 mo. 5 da.	8%
6.	\$12875	3 mo. 9 da.	$4\frac{1}{2}$%
7.	\$4560	4 yr. 2 mo. 18 da.	5%
8.	\$125	1 mo. 12 da.	7%

51. The six per cent method. We shall now find a very simple and convenient method for finding interest at six per cent. This method is also useful for finding interest at other rates.

1. What is the interest on \$1 at 6% for 1 year? For 1 month? For 1 day?

2. Find the interest on \$1 at 6% for 2 years; 5 years; 12 years; 6 years.

3. Find the interest on \$1 at 6% for 4 months; 9 months; 10 months; 5 months.

4. Find the interest on \$1 at 6% for 12 days; 18 days; 15 days; 20 days; 5 days.

5. Make a rule for finding the interest on \$1 at 6% for any number of years; for any number of months; for any number of days.

The answer to these questions make clear the following

Rule. *To find the interest on* \$1 *at* 6% *for any number of years, months, and days, take 6¢ for every year, ½¢ for every month, and* $\frac{1}{6}$ *of a mill for every day.*

Exercise 54

Find the interest on \$1 at 6% for the following periods:

1. 2 yr. 8 mo. 12 da.

SOLUTION. \$.12 + \$.04 + \$.002 = \$.162.

2. 3 yr. 2 mo. 18 da.
3. 1 yr. 5 mo. 10 da.
4. 4 yr. 3 mo. 15 da.
5. 7 mo. 20 da.
6. 6 mo. 8 da.
7. 1 yr. 1 mo. 1 da.
8. 25 da.
9. 5 yr. 25 da.
10. 3 yr. 3 mo. 12 da.
11. 2 yr. 15 da.
12. 27 da.
13. 5 mo. 2 da.
14. 9 mo. 20 da.
15. 120 da.

16. Find the interest on \$450 at 6% for 3 yr. 7 mo. 10 da.

SOLUTION. The interest on \$1 at 6% for 3 yr. 7 mo. 10 da. $=\$.18+\$.035+\$.001\frac{2}{3}$
$=\$.216\frac{2}{3}$.

Then the interest on \$450 $=450\times\$.216\frac{2}{3}$
$=\$97.50$

.18	450
.035	$.216\frac{2}{3}$
$.001\frac{2}{3}$	300
$.216\frac{2}{3}$	2700
	450
	900
	97.500

Find the interest at 6% on

17. \$100 for 1 yr. 6 mo. 24 da.

18. \$600 for 3 yr. 7 mo.

19. \$3000 for 1 yr. 1 mo. 1 da.

20. \$375 for 9 mo. 20 da.

21. \$72.50 for 4 yr. 3 mo. 9 da.

22. \$8967.50 for 1 yr. 4 mo. 13 da.

23. \$753.25 for 2 mo. 10 da.

52. Interest at any given rate by the six per cent method. If the interest on a given sum of money for a given time at 6% is \$300, what is the interest at 3%? At 4%? At 5%? At 7%?

EXAMPLE. Find the interest on \$1250 for 2 yr. 5 mo. 14 da. at 7%.

SOLUTION. The interest on \$1 for 2 yr. 5 mo. 14 da. at 6% $=\$.147\frac{1}{3}$.

The interest on \$1250 for 2 yr. 5 mo. 14 da. at 6% $=1250\times\$.147\frac{1}{3}$
$=\$184.167$.

The interest on \$1250 for 2 yr. 5 mo. 14 da. at 7% $=1\frac{1}{6}\times\$184.167$
$=\$214.86$.

\$.12	1250
.025	$.147\frac{1}{3}$
$.002\frac{1}{3}$	417
$.147\frac{1}{3}$	8750
	5000
	1250
	6)184.167
	30.695
	214.862

Exercise 55

Find the interest on

1. \$50 for 3 yr. 5 mo. at 7%.
2. \$400 for 7 mo. 12 da. at 5%; at 2%; at 4%.
3. \$25.50 for 1 yr. 6 mo. 15 da. at 3%; at 7%.
4. \$92.40 for 8 mo. 23 da. at $4\frac{1}{2}$%; at 5%.
5. \$3000 for 25 da. at 8%.
6. \$840 for 2 yr. 24 da. at 5%.
7. \$1275.25 for 5 mo. at 4%.

53. Finding the time between two dates. The time for which interest is to be paid is found by computing the time from the date on which the loan was made to the date on which it was paid. This is usually done by subtracting the dates, considering each month to have 30 days.

EXAMPLE. For what time should interest be paid on a loan made August 10, 1918, and paid December 4, 1920?

SOLUTION. August is the eighth month and December is the twelfth month.

	yr.	mo.	da.
From	1920	12	4
take	1918	8	10
	2	3	24

Banks, whose loans are usually made for only a short time, count the exact number of days between the dates.

EXAMPLE. Find the time from January 6, 1916, to June 19, 1916.

SOLUTION. There are remaining in January 25 da., February 29 da., March 31 da., April 30 da., May 31 da., June 19 da.

The total number of days is 165.

25
29
31
30
31
19
165

Exercise 56

In the following four exercises find the difference between the two dates by subtraction :

1. January 1, 1919, to May 7, 1920.
2. October 12, 1918, to November 28, 1921.
3. May 23, 1917, to February 5, 1919.
4. August 17, 1918, to May 20, 1920.

In the next six exercises find the exact number of days:

5. August 7 to October 21.
6. May 10 to July 14.
7. September 14 to February 19, 1918.
8. February 8 to September 13, 1919.
9. December 12, 1919, to August 6, 1920.
10. May 31 to November 1.
11. Find the interest on \$800 at 6% for the exact number of days from June 10 to October 22.
12. Find the interest on \$1200 at 6% for the exact number of days from May 12 to July 25.

In the remaining exercises find the time by subtracting dates.

13. Find the interest on \$875.50 at 7% from January 2, 1912, to October 1, 1915.
14. Find the interest on \$680 at 5% from December 19, 1900, to April 5, 1904.
15. Find the interest on \$4500 from May 10, 1919, to July 6, 1920, at 7%.
16. Find the interest on \$90 from Oct. 23, 1919, to March 6, 1921, at 6%.
17. *Find the amount of \$750 from Feb. 3 to Nov. 17 at 5%.*

54. Promissory notes. When one person borrows money from another the borrower usually gives the lender a written statement of their agreement.

Such a statement is called a **promissory note.**

The following is one form of a promissory note :

> $285.$\frac{40}{}$ *Frankville, Idaho, Nov. 1, 1918.*
>
> *Three years after date..I..promise to pay to the order of William Jameson*--------------------------
>
> *Two Hundred Eighty-five and* $\frac{40}{100}$ *Dollars with interest at 5% per annum from date, for value received.*
>
> *Robert H. Axtell.*

A promissory note is a written promise to pay a certain sum of money at a certain time.

In this note Robert H. Axtell is the **maker**, William Jameson is the **payee**, and $285.40 is the **face** or **principal**.

The date a note is due is called the **date of maturity.**

> $450.$\frac{00}{100}$ *Macon, Georgia, May 12, 1919.*
>
> *On demand, I promise to pay to the order of Arthur M. Davis*--------------------------
>
> *Four Hundred Fifty and* $\frac{00}{100}$ *Dollars for value received.*
>
> *Hugo McKay.*

A note should state the date, face, the promise to pay, *payee*, maker, and time to run. If the note bears interest, as *in the above* example, the rate of interest should be named.

The second of the above notes is an example of a **non-interest bearing, demand note.**

Exercise 57

1. Who must pay the first of the above notes when it becomes due? Who is to receive the money? How much is to be paid?

2. Write a note having James Gallagher as payee, William Hawley as maker, principal \$500, rate of interest 6%, time 30 days. Compute the interest and the sum the payee receives when the note is due.

3. Suppose that you buy a horse for \$135 from one of your classmates and that, instead of paying cash, you give your note due in six months and bearing 7% interest. Write the note which you would give and compute the amount to be paid.

4. Who is the payee of the note of example 3? Who is the maker? Who writes the name of the payee? Of the maker? In how many places on the note is the amount to be paid written? What is the meaning of the words "for value received" in a note?

Write notes for the following and compute the amount to be paid to the payee.

5. Principal, \$250 ; time, 90 days ; maker, William Jennings ; payee, John K. Adams ; rate, 6%.

6. Principal, \$4500 ; time to be paid, Jan. 1, 1927 ; time the note is dated, Dec. 1, 1925; payee, A. N. Collins ; maker, Arthur Donley ; rate, $5\frac{1}{2}$%.

7. Write a demand note payable to yourself for the sum of \$1500 with George Marsh as maker.

8. Who holds each of the above notes? Who pays each? *Who gets each note* after it is paid?

55. To find the principal, rate, or time.

$30 = 2 \times 3 \times$ what number? $54 = 3 \times 3 \times$ what number?
$63 = 3 \times 7 \times$ what number? $105 = 5 \times 3 \times$ what number?

If you are told the product of three factors and two of the factors, how can you find the third factor?

Three numbers multiplied together give 90. Two of them are 6 and 3. What is the other?

$\$12 = \$100 \times .06 \times ?$ $\$14.50 = \$100 \times .07 \times ?$
$\$15 = \$? \times .06 \times 5.$ $\$47.25 = \$? \times .07 \times 1\frac{1}{2}.$

The interest formula tells you that the interest is the product of three factors—principal, rate, and time. If you are told the interest on a certain principal at a certain rate, how can you find the time?

Make a formula for finding the time when the interest, principal, and rate are given.

Make a formula for finding the rate when the interest, principal, and time are given.

Make a formula for finding the principal when the interest, rate, and time are given.

Exercise 58

Find the missing numbers below, using these formulas:

$$p = \frac{i}{rt}; \; r = \frac{i}{pt}; \; t = \frac{i}{pr}; \; i = prt.$$

	PRINCIPAL	TIME	RATE	INTEREST
1.	$320	?	6%	$12.80

SOLUTION. $t = \frac{i}{pr} = \frac{\overset{.04}{\cancel{12.80}}}{\cancel{320} \times .06} = \frac{2}{3}$

The time is therefore $\frac{2}{3}$ yr. or 8 mo.

2.	?	9 mo.	6%	$ 7.88
3.	$185	2 yr. 6 mo.	?	$23.13
4.	$480	1 yr. 6 mo.	5%	?
5.	?	1 yr. 3 mo.	$3\frac{1}{2}$%	$10.50
6.	*$275.25*	?	4%	$55.05
7.	*$175*	2 yr. 4 mo. 3 da.	?	$24.59

Exercise 59. Review of Interest

Solve as many of these examples without pencil as you can.

1. What is the interest on \$100 for 1 year at 3%? On \$100 for 2 years at 6%? On \$600 for six months at 8%?

2. Find the interest on \$200 for 12 days at 6% ; on \$500 for 15 days at 6% ; on \$450 for 2 months 18 days at 6%.

3. State the formula for finding interest.

4. If you know the interest on a sum of money for 1 year, how can you find the interest for 3 years? For 6 months? For 1 month? For 10 days?

5. If you know the interest on a sum of money for a certain time at 4%, how can you find the interest for the same time at 2%? At 3%? At 8%? At 5%?

6. Express as days, 1 yr. 1 mo. 1 da. ; 7 mo. 10 da. ; 2 yr. 5 mo. 12 da.

7. Express each of the time periods of the preceding example as months ; as years.

Solve each of the next three examples by the formula method, then by the six per cent method, and decide which is the easier for each example.

8. Find the interest on \$240 at 8% for 1 yr. 8 mo. 12 da.

9. Find the interest on \$25 at 5% for 5 mo. 10 da.

10. Find the interest on \$4980 at 7% for 2 yr. 10 mo. 11 da.

11. Find the time between June 17, 1775, and July 4, 1776, by subtracting the dates.

12. Find the exact number of days between August 1, 1922, and May 1, 1923.

13. A farmer's net income from his farm of 80 acres is \$575. At what price per acre shall his farm be valued so that this income shall be 5% interest on the value of his farm?

14. *In estimating* the cost of growing an acre of wheat, an *allowance of \$4.464* was made for interest and taxes. The

land was valued at $62 an acre. One and one-half per cent of the value was allowed for the taxes. What per cent was allowed for interest?

15. You are offered a house and lot for $4200. The house is rented for $30 a month. The repairs will probably amount to $40 a year and other costs to $35 a year. If you buy it, what per cent per annum will it yield on your investment?

16. A Liberty Bond is equivalent to a promissory note made by the United States Government. These bonds bear $4\frac{1}{4}\%$ interest per annum on the face of the bond, which is the price at which they were sold by the Government. What is Mr. Snyder's income on the 15 fifty-dollar bonds which he owns?

17. A real-estate dealer bought a lot for $750 on April 12, and sold it on December 23 for $900. On the day he sold this first lot he bought a second lot for the $900. This lot he sold on the following June 12 for $1100. What rate of interest per annum did he make on his original sum of money?

18. A retail merchant bought a bill of goods for $1275. If he had paid cash, he would have been allowed a discount of 3%. Instead, he gave his note for the $1275 bearing 6% interest to run 9 months. How much would he have saved by borrowing the money needed to pay cash for the bill, paying 7% interest per annum for the 9 months?

19. By buying his coal in July Mr. Barclay paid $5.35 a ton for it. If he had waited until January to buy it, he must have paid $7.15 a ton for it. What rate of interest on his money did he get by buying in July?

20. A certain piece of furniture may be bought for $36 cash or on the installment plan for $3.50 at the end of each month for 12 months. Not considering interest due on the installments paid, what rate of interest on the cash price is being paid by one who buys this furniture on the installment plan?

CHAPTER VII

MEASUREMENT OF LINES

56. Line segments. The part of the straight line between the points A and B in this figure is called a **line segment**, or simply a segment. It is read AB.

FIG. 2.

A line segment is sometimes referred to as a line.

The **ruler**, or other **straightedge**, is used in drawing straight lines. The ruler is also used in measuring distances.

The **compasses** are used in drawing circles and in marking off lines equal to given lines.

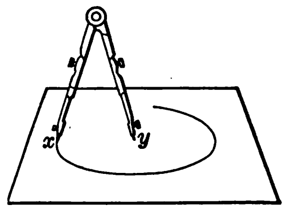


FIG. 3.

Exercise 60

1. Draw a line segment equal to the line segment AB in Figure 2.

   HINT. Measure the length of AB on a ruler and then draw a line segment of the same length.

2. Draw a segment whose length equals the length of this page.

3. Draw a segment $3\frac{5}{16}$ in. long.

4. Open the compasses a distance of 1 in. as marked on the ruler. This means that the feet of the compasses, x and y in Figure 3, shall be 1 in. apart.

5. Open the compasses a distance of 2 in.; of $\frac{3}{4}$ in.; of *AB, Figure 2.*

6. Draw a segment equal to AB, Figure 2, by using the compasses.

SOLUTION. From a point M draw a line MX at least as long as AB. Open the compasses the distance AB. Mark off from M on MX the distance MN equal to AB. Then MN is the required line segment.

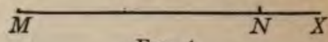


FIG. 4.

7. Draw two line segments of different lengths. By using the compasses make two segments equal to these segments.

8. Draw a line segment. Draw a segment equal to it, first by using the ruler only, and then by using ruler and compasses. Which method is probably more accurate?

9. How many straight lines can be drawn through a given point P? How many straight lines can be drawn so as to pass through both of two given points A and B? Can you always draw a straight line that will pass through any three given points X, Y, and Z?

NOTE. To do the work of the remaining chapters in this book each pupil should be supplied with the following materials: a hard pencil and a medium pencil, both well sharpened; six-inch celluloid ruler and foot-ruler; compasses, protractor, and eraser; loose-leaf notebook, with unruled paper.

Suggestions for the teacher. Constructions should be made in class at first.

Insist upon neatness and as high a degree of accuracy as may be expected of pupils of this grade.

Do not accept careless and inaccurate work. Have the constructions repeated until they are done satisfactorily.

The constructions should be made in permanent notebooks.

The figures should be made in fine lines with a hard pencil.

Each construction in the notebook should be numbered for convenience in grading.

Much of the examination of notebooks may be made during the recitation period while the figures are being drawn.

The room should be provided with a blackboard protractor.

10. The point A is called a **corner** or **vertex** of the cube. How many vertices has the cube?

11. The line AB is called an **edge** of the cube. How many edges has a cube?

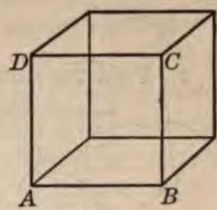


FIG. 5.

12. What is the sum of the edges of a cube if one edge is 4 in.? If one edge is e units long? Make a formula for finding the sum of the edges of a cube.

13. Find the length of one edge of a cube if the sum of the edges is 40 in. Make a formula for finding one edge, e, when the sum of the edges, s, is known.

14. A box is 4 ft. long, 18 in. high, and $2\frac{1}{2}$ ft. wide. When the box is nailed up for shipping a metal strip is nailed along each edge to make the box stronger. How many feet of such metal strip will be required to strengthen 100 boxes in this way?

15. The point O is called the **vertex** of the **pyramid**. The line OA is called a **lateral edge** of the pyramid. $ABCD$ is the **base** of the pyramid. How many edges has this pyramid? If the base is a 6-inch square, and each lateral edge is 8 in., what is the sum of the edges of this pyramid? If the length of each edge of the base is e and the length of each lateral edge is l, make a formula for finding the sum of the edges.

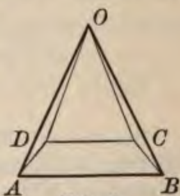


FIG. 6.

57. Finding the sum and the difference of line segments. In making drawings it is often necessary to draw a line equal to the combined lengths of two or more lines. The sum and also the difference of line segments may be found by using *the compasses and* without measuring the segments.

Exercise 61

1. Find the sum of the segments, a, b, and c.

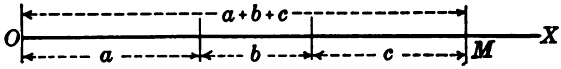


FIG. 7.

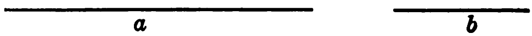
SOLUTION. Draw a line OX of indefinite length from the point O. Then open the compasses a distance a, and lay off from the point O a segment equal to a. Then lay off b and c in a similar way, as in the figure. The segment OM is the sum of a, b, and c. We may write

$$OM = a + b + c.$$

2. Draw two line segments and find their sum.

3. Draw a triangle. Draw a line equal to the sum of its sides.

4. Draw a line segment equal to the difference of the given segments a and b.



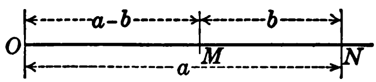


FIG. 8.

SOLUTION. From a point O draw a line at least as long as a. Lay off from O a segment ON equal to a. Then from N lay off toward O a segment NM equal to b. The segment OM is the difference between a and b. We may write

$$OM = a - b.$$

5. Draw three line segments a, b, and c. From the sum of a and b take c.

6. Draw a triangle. Subtract the longest side from the *sum of the other* two sides.

58. Accuracy in measuring. Let each of three pupils measure the length of the schoolroom with a foot-ruler and compare the results.

In applying a ruler you place one end on a mark previously made and mark the position of the other end. Errors are probable in doing both these things. The ruler may be a little too long or a little too short. For such reasons as these no measurement can be known to be entirely accurate. It is very important for a pupil to know how to make his errors in measuring as small as possible, and to know what degree of accuracy may be expected under given conditions.

Exercise 62

1. In measuring with a foot-ruler you place one end on a mark previously made and mark the position of the other end. In doing either of these things can you be sure that you will not make an error of $\frac{1}{2}$ in.? Of $\frac{1}{4}$ in.? Of $\frac{1}{8}$ in.? Of $\frac{1}{16}$ in.? Of $\frac{1}{32}$ in.? Of .01 in.? Of .001 in.? We will suppose that the divisions are accurately marked on your ruler, that it has square ends, and that you have a well-sharpened pencil.

2. A boy's height is marked on the wall. It is measured by each of 4 pupils. The results are 4 ft. $3\frac{1}{2}$ in., 4 ft. 3 in., 4 ft. $3\frac{3}{4}$ in., and 4 ft. $3\frac{5}{8}$ in. What is the greatest difference between any two measurements? Do you know the boy's height? Are these results probably accurate within 1 in.? Within $\frac{1}{2}$ in.? Within $\frac{1}{4}$ in.?

3. Let each pupil measure the length of this page as accurately as possible. Write the results on the blackboard. Do you know the length of the page within 1 in.? Within .01 in.?

4. Let each pupil measure the length of the second line of print on this page. Compare the results. What is the *greatest difference* between any two?

5. Mark the height of a pupil on the blackboard. Let each of 4 pupils measure it. What is the greatest difference between any two measurements? How accurately do you know this pupil's height, that is, to what fraction of an inch?

6. If your height is marked on the blackboard, about how accurately can you measure it with a foot-ruler?

7. About how accurately can you measure the length of your schoolroom with a foot-ruler? Let each of 6 pupils measure it. Are you sure of the length within 1 ft.? Within 1 in.? Within $\frac{1}{2}$ in.? Let each of 6 pupils measure it with a yardstick. What degree of accuracy do you think you now have?

8. You have probably concluded that measuring cannot be expected to be exact. When we say that we have measured a board and that it is 16 ft. long, we do not mean that we know that the length is exactly 16 ft., but that the length differs from 16 ft. by a small amount. This amount will depend upon care in measuring, accuracy of the measuring instruments, and the length of the distance measured.

If a boy does not make an error of more than $\frac{1}{8}$ in. in each application of a foot-ruler, what is the largest error that he can make in measuring a line that is between 11 and 12 ft. long? If he does not make an error of more than $\frac{1}{8}$ in. in each application of a yardstick, what is the largest error that he can make in measuring the same line with a yardstick?

9. John is making some bookshelves. The shelves are between 3 ft. and 4 ft. long, and he is measuring with a foot-ruler. To have a fairly good job he must not make an error of more than $\frac{1}{4}$ in. in the length of a shelf. Can you measure as accurately as that?

10. Suppose that an error of 3 in. is made in measuring a length known to be 20 ft. What is the per cent of error? If the same per cent of error is made in measuring a mile, how many feet does it amount to?

11. An error of 1% amounts to how much in 10 ft.? Can you measure 10 ft. as accurately as that with a foot-ruler? With a yardstick?

12. The United States Coast and Geodetic Survey measured a line from Cape May, New Jersey, to Point Arenas, California, and found the distance to be 2625 miles. It is thought that the error is not more than 100 ft. What per cent of error is that?

13. If a distance is measured several times, some results will probably be too large and some too small. Suppose that in measuring a distance afterwards found to be 10 ft. one boy gets as his result 9 ft. 11 in., and another gets 10 ft. $1\frac{1}{2}$ in. The sum of these is 20 ft. $\frac{1}{2}$ in. One-half of this sum, the average of the two, is 10 ft. $\frac{1}{4}$ in. In this case the average is nearer the correct distance than the result found by either boy.

This need not be the case, however. If one boy had found the distance to be 10 ft. 1 in., and the other 10 ft. $1\frac{1}{2}$ in., then the average, 10 ft. $1\frac{1}{4}$ in., would not be as near the correct distance as 10 ft. 1 in. But generally some of the measurements will give results that are too large, and others too small. So it is usually better to depend on the average of a number of measurements than upon a single one. Hence where a high degree of accuracy is desired, it is common to make several measurements and to take their average.

What is the average of the results in exercise 2?

14. What is the average of the results in exercise 5? If this result is taken to be the height of the pupil, what per cent of error did each pupil make?

15. What is the average of the results in exercise 3?

16. With a ruler draw the following line segments: $AB = 1\frac{1}{2}$ in.; $CD = \frac{3}{4}$ in.; $EF = \frac{9}{16}$ in.; $GH = \frac{5}{8}$ in. Using the compasses, find the sum of these segments. Measure the line which gives the sum. How long do you find it to be? How long should it be?

59. Drawing to scale. House plans and maps are examples of drawings made to scale. The figure represents the floor plan of a summer cottage. It is drawn to the scale of 16 ft. to 1 in.; that is, 16 ft. are represented by 1 in.

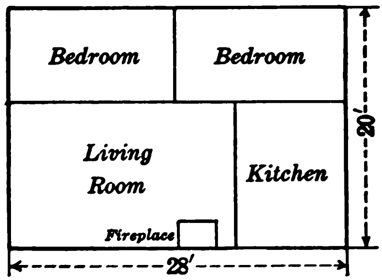


FIG. 9.

Exercise 63

1. On the scale of 400 miles to 1 inch, what distance represents 800 mi.? 1200 mi.? 1000 mi.? 100 mi.? 50 mi.? 10 mi.?

2. On the scale of 16 ft. to 1 in., what represents 32 ft.? 40 ft.? 10 ft.? 8 ft.? 1 ft.?

3. On the scale of 400 mi. to 1 in., what distance does 2 in. represent? $\frac{1}{2}$ in.? $\frac{1}{4}$ in.? $\frac{1}{8}$ in.? $\frac{1}{16}$ in.?

4. On the scale of 16 ft. to 1 in., what does 5 in. represent? $3\frac{1}{2}$ in.? $\frac{1}{2}$ in.? $\frac{1}{8}$ in.? $\frac{1}{16}$ in.?

5. On the scale of 400 mi. to 1 in., what represents 1000 mi.? 6000 mi.? 2650 mi.? 960 mi.?

6. A schoolroom floor is 30 ft. by 40 ft. Make a drawing of it on the scale of 10 ft. to 1 in. Use the end of your ruler to make square corners.

7. If you had wanted this drawing to be $2\frac{1}{2}$ in. long, what sc[illegible] have used?

If your ruler is marked to sixteenths of an inch, you should be able to draw a line of given length, not longer than your ruler, and not make an error of more than $\frac{1}{16}$ in. But you cannot be certain to avoid an error of $\frac{1}{32}$ in. Hence in the following exercises get the answers correct to the nearest $\frac{1}{16}$ in.

8. On the scale of 400 mi. to 1 in., what represents 987 mi.?

SOLUTION. Since we wish the answer correct to $\frac{1}{16}$ in., we will first find what distance is represented by $\frac{1}{16}$ in. Since 1 in. represents 400 miles, $\frac{1}{16}$ in. represents 25 mi. To find how many sixteenths of an inch are required to represent 987 mi., divide 987 by 25. $987 \div 25 = 39$, to the nearest unit. Hence 987 mi. are represented by $\frac{39}{16}$ in., which equals $2\frac{7}{16}$ in.

9. On the scale of 400 mi. to 1 in., what represents 2000 mi.? 1640 mi.? 365 mi.? 227 mi.?

10. On the scale of 20 ft. to 1 in., what represents 65 ft.? 141 ft.? 346 ft.? 17 ft.?

11. The scale in the drawing of the cottage floor plan given above is 16 ft. to 1 in. Find the dimensions of the living room, bedrooms, kitchen, and fireplace.

12. The floor of a room is 20 ft. $\times$ 16 ft. Make a drawing of it on the scale of 8 ft. to 1 in. To make square corners use the end of your ruler or the corner of a sheet of paper. A space on the floor 3 ft.$\times$4 ft. is occupied by a desk. The desk is near one corner of the room, 6 ft. from one side and 8 ft. from one end. Locate this desk in your drawing.

13. Make a drawing of the cottage floor plan given above on the scale of 4 ft. to 1 in.

14. Measure the floor of your schoolroom and make a drawing of it on a convenient scale.

15. *What is the scale of the map of the United States in your geography?* Of North America? Of your state?

Graphs

60. Representing numbers by lines. On certain direct railroad lines the distance from New York to Chicago is 909 mi.; from Chicago to Denver, 1034 mi. ; and from Denver to San Francisco, 1668 mi. These distances are more easily compared when represented by straight lines. In Figure 10 the scale is 640 miles to 1 in.

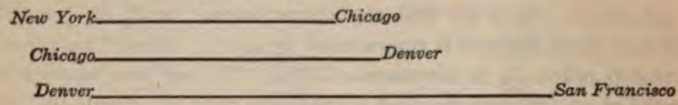


Fig. 10.

A straight line that represents a number is called a **graph** of that number. The representation of numbers by lines is called **graphic representation.**

Many discussions involving statistics in newspapers, magazines, and books are illustrated by graphs. The graph serves to make the statistics more easily and more clearly understood.

Exercise 64

1. Using the distances given above, draw lines to represent the distance from New York to Chicago, from New York to Denver, and from New York to San Francisco. Use the scale of 80 mi. to $\frac{1}{16}$ in.

Solution. First make the following table:

Distance in Miles		Distance Reduced to Scale
New York to Chicago . .	909 mi.	$\frac{11}{16}$ in.
New York to Denver . . .	1943 mi.	$1\frac{1}{2}$ in.
New York to San Francisco .	3611 mi.	$2\frac{13}{16}$ in.

$$80\overline{)909}\ \ 11$$

$$80\overline{)1943}\ \ 24$$

$$80\overline{)3611}\ \ 45$$

In reducing these distances to scale we wish the answers correct *to the nearest* [illegible] These answers are given in the divisions at

the right. A quotient such as 11 means that 909 mi. are represented by $\frac{11}{16}$ in.

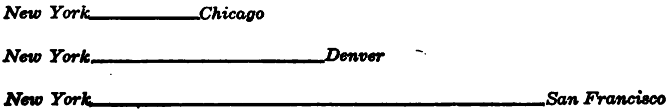


FIG. 11.

In solving the following problems the pupil should make a table like the one given above.

2. Represent by straight lines on the scale of 400 mi. to 1 in. the following distances: Chicago to New Orleans, 930 mi.; Chicago to Duluth, 608 mi.; Duluth to New Orleans, 1538 mi.; Seattle to Los Angeles, 1430 mi.

3. Represent on the blackboard the distances in the last two exercises on the scale of 1000 mi. to 1 ft.

4. The heights of the highest mountains on the different continents are as follows:

CONTINENT	MOUNTAIN	HEIGHT IN FEET
Australia	Mt. Kosciusko	7,328
Europe	Mt. Blanc	15,782
Africa	Kibo Peak	19,320
North America	Mt. McKinley	20,300
South America	Mt. Aconcagua	23,080
Asia	Mt. Everest	29,002

The comparison of these heights becomes more striking when they are represented by vertical lines. The scale used *here is 20,000 ft. to 1 in.*

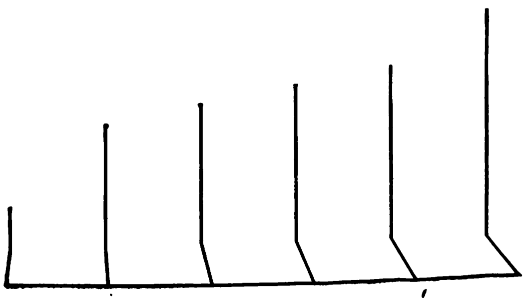


FIG. 12.

Let the pupil make a similar drawing on the scale of 12,000 ft. to 1 in.

5. The distances from the sun to the planets in millions of miles are as follows :

Planet	Distance in Millions of Miles	Planet	Distance in Millions of Miles
Mercury . . .	36	Jupiter	483
Venus	67	Saturn	886
Earth	93	Uranus	1782
Mars	142	Neptune . . .	2792

Represent these distances graphically on the scale of twenty million miles to $\frac{1}{16}$ of an inch.

6. The following tables give the average temperature and the rainfall in Charleston, Illinois, for a certain year. The temperature is given in degrees and the rainfall in inches. Represent the temperature graphically on the scale of 16° to 1 in. Represent the rainfall graphically, using the numbers of inches in the table. Find the average monthly rainfall, represent it graphically, and notice how each month's rainfall varies from the average.

	Temperature	Rainfall
January . .	33.1	9.37
February .	29.3	1.03
March . .	40.3	1.45
April . . .	50.8	1.18
May . . .	65.6	3.70
June . . .	68.2	4.61
July . . .	80.7	1.38
August . .	77.7	2.65
September .	65.6	3.34
October . .	55.2	2.10
November .	45.8	1.88
December .	29.7	4.03

7. The number of miles of railway in the United States in 1870 was 52,922; in 1880, 93,267; in 1890, 167,191; in 1900, 198,964. Write these numbers to the nearest 1000 and represent them graphically using $\frac{1}{16}$ in. to 3000 mi.

CIRCLES

61. Some definitions. A flat, smooth surface is called a **plane surface,** or simply a **plane.**

The surface of the blackboard, the top of the table, and the floor may be taken as examples of a plane.

A closed curve lying in a plane and such that all of its points are equally distant from a point in the plane is called a **circle.**

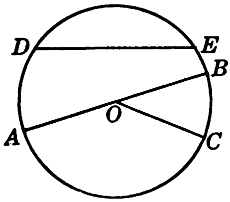


FIG. 13.

The point in the plane from which all of the points in the circle are equally distant is called the **center.**

A line from the center to the circle is called the **radius.**

A line through the center of the circle terminating at both ends in the circumference is called the **diameter.**

From the way in which a circle is drawn it is clear that *all radii are equal* and that *all diameters are equal.*

A line joining two points of a circle is called a **chord.**

The length of the circle is called the **circumference.**

Any portion of a circle is called an **arc.**

A **semicircle** is one-half of a circle. A **quadrant** is one-fourth of a circle.

In Figure 13, O is the center, AB is a diameter, OC is a radius, BC is an arc, and DE is a chord.

An arc which is $\frac{1}{360}$ of a circle is called a **degree** of arc.

Circles are drawn with compasses.

Exercise 65

1. With your compasses draw a circle with a radius of 2 in.

2. In how many points may two straight lines intersect? *In how many points may two circles intersect? Draw two circles that intersect.*

3. How many degrees in a semicircle? In a quadrant?

4. Over how many degrees of arc does the end of the minute hand of a clock pass in 1 hour? In 15 min.? In 30 min.? In 45 min.? In 5 min.? In 10 min.? In 50 min.?

5. How many degrees of arc are there from the equator to the north pole?

6. A diameter of a circle divides the circle into how many arcs? How many degrees in each arc?

7. What is the longest chord that can be drawn in a circle?

8. Is 1° of arc of a circle of radius 2 in. the same length as 1° of arc of a circle of radius 5 in.? Which is longer, 1° of arc on the arctic circle or 1° of arc on the equator?

9. Name two kinds of units in which an arc of a circle may be measured.

10. If a circle is 10 in. long, how long is its semicircle? Its quadrant?

11. At the entrance to a harbor are two revolving guns, 8 miles apart. One has a range of 6 miles and the other a range of 5 miles. Make a drawing to show the surface over which each gun can shoot, and the surface that can be reached by both guns.

12. The radii of the moon, earth, and sun are approximately 1080 mi., 3960 mi., and 433,000 mi. If the moon is represented by a circle of radius 1 in., what should be the radii of circles to represent the earth and the sun, on the same scale?

13. Choose a convenient scale and draw on the blackboard circles to represent the moon, earth, and sun.

14. Make a figure like Figure 14. Use a radius of 2 in. *The radius of the circle is used in making the arcs that divide the circle into six equal parts.*

15. Make a six-pointed star. Find the points of the star by drawing arcs as in the last exercise. See Figure 15.

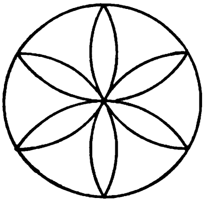


FIG. 14.

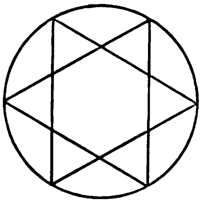


FIG. 15.

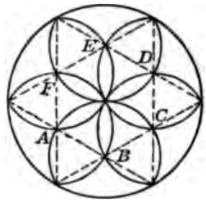


FIG. 16.

16. Make Figure 16. Divide the outer circle into equal parts as in exercise 14. Mark the six-pointed star in light lines. Use points A, B, C, D, E, and F as the centers of the small circles.

17. Find geometric figures used in decoration, as in church windows, in ironwork or woodwork, or in patterns in wall paper, linoleum, or carpets that use the forms given in Figures 14, 15, and 16.

62. The relation between the circumference and the diameter of a circle. In many problems there is given either the circumference or the diameter of a circle to find the other. In trying to find out what the diameter of a circle must be multiplied by to get the circumference a pupil made the following measurements :

OBJECT	CIRCUMFERENCE	DIAMETER
Piano stool	$46\frac{1}{4}$ in.	$14\frac{5}{8}$ in.
Tin pail	30 in.	$9\frac{1}{2}$ in.
Flower pot	$21\frac{1}{8}$ in.	$6\frac{3}{4}$ in.
Lamp shade	$31\frac{9}{16}$ in.	$10\frac{1}{16}$ in.

She then divided each circumference by its diameter and *compared the* results.

1. Divide each of the above circumferences by its diameter. Find the quotient correct to .01. Find the average of these quotients.

2. Measure the circumference and the diameter of each of four large circular objects. Divide each circumference by its diameter. Find the average of the quotients.

3. It is known that the quotient obtained by dividing the circumference of a circle by its diameter is the same for all circles. Do you see any reason why the quotients you have obtained are not all equal?

4. Cut from cardboard a circle of radius 4 in. Mark a point P on this circle. Lay a sheet of paper on your desk and roll the circle along a straight line on this sheet of paper. Measure the distance between two successive points where P touches the sheet of paper. This distance is how many times the diameter?

5. It is shown in geometry that in any circle the circumference is about $3\frac{1}{7}$ times the diameter, or more accurately, about 3.1416 times the diameter. This number by which the diameter must be multiplied to get the circumference is represented by the Greek letter π (pronounced pī).

If the diameter d of a circle is known, the circumference is obtained by the formula $c=\pi d$.

Let the pupil show that if the radius r is known the circumference is obtained by the formula $c=2\pi r$.

Exercise 66

1. Find the circumference of a circle whose diameter is 21 ft.; 14 in.; 65 in.; 3.43 ft.; $2\frac{1}{2}$ in.; $\frac{5}{6}$ in. Use $\pi=3\frac{1}{7}$.

2. Find the circumference of a circle whose radius is 10 ft.; 3 miles; 4000 miles; 46.3 ft. Use $\pi=3.1416$.

3. Find the diameter of a circle whose circumference is 22 in.; $31\frac{3}{7}$ in.; 96 in.; 100 in. Use $\pi=3\frac{1}{7}$.

4. Find the radius of a circle whose circumference is 314.16 ft.; 204.204 ft.; 1000 ft.; 4763 miles. Use $\pi = 3.1416$. Find each answer correct to .01.

5. Make a rule for finding the diameter of a circle when the circumference is given. Write this rule as a formula.

6. Make a rule for finding the radius of a circle when the circumference is given. Write this rule as a formula.

7. The value of π correct to 8 decimal places is 3.14159625. What is the value correct to 7 decimal places? To 6 decimal places? To 5 decimal places? To 4 decimal places?

8. Reduce $3\frac{1}{7}$ to a decimal fraction correct to .0001. To how many decimal places is this the correct value of π?

9. Which value of π is more convenient to use in solving problems, $3\frac{1}{7}$ or 3.1416? Can you give an example in which $3\frac{1}{7}$ is more convenient? Can you give an example in which it is more convenient to use 3.1416? Which value is more accurate?

10. Suppose that in a problem you wish to compute the circumference of a circle correct to .1 ft. Which value of π should be chosen if the diameter is 1 ft.? If the diameter is 10 ft.? 100 ft.? 1000 ft.? 20 ft.? 35 ft.? 982 ft.? Compute the circumference in each case using both values of π, and compare the results. Which value of π should be used when the diameter is a large number?

These examples show that for small circles the value $3\frac{1}{7}$ gives results that are sufficiently accurate for most purposes. But when a higher degree of accuracy is desired the value 3.1416 must be used. In the problems in this book the value 3.1416 is to be used unless otherwise stated.

11. The diameter of the earth at the equator is 7918 mi. Find its circumference correct to 1 mi.

12. The diameter of the sun is 866,400 mi. Find its circumference correct to 100 mi.

13. The diameter of the wheel of a bicycle is 28 in. How many revolutions will the wheel make in going 1 mi.? Answer correct to .1. The wheel of this bicycle makes $2\frac{3}{7}$ revolutions for each revolution of the pedal. How many revolutions of the pedal will be made in going around a mile track?

14. A wire is $\frac{1}{16}$ in. in diameter. Find its circumference correct to .001 in.

15. The flywheel of an engine is 4 ft. in diameter. The wheel makes 1000 revolutions in a minute. How far does a point on the rim of the wheel travel in a minute? In a second? Answers correct to .1 ft.

16. The fore wheel of a wagon is 42 in. in diameter, and the hind wheel 49 in. How many more revolutions does the fore wheel make than the hind wheel in going a mile?

17. A merry-go-round is 60 ft. in diameter. John rides around 20 times for a 5-cent fare. How far does he ride? At the same rate how much should he pay to ride a mile?

18. Two boys run side by side, 3 ft. apart, around a circular track. The one on the inside runs $\frac{1}{4}$ mile. How far does the other run?

19. An automobile goes around a circular track. The inside wheels travel 1 mi. The distance between the inside and the outside wheels is 56 in. How far do the outside wheels travel? A wheel is 34 in. in diameter. How many times do the inside wheels turn? How many times do the outside wheels turn?

20. Two automobiles race for a quarter of a mile. One has 32-inch wheels and the other 36-inch wheels. How many *more times* do the smaller wheels turn than the larger in going *that distance?*

Exercise 67. Miscellaneous Problems

1. Given two lines a and b, construct $2a + 3b$.

2. Given a rectangle, construct a line equal to its perimeter.

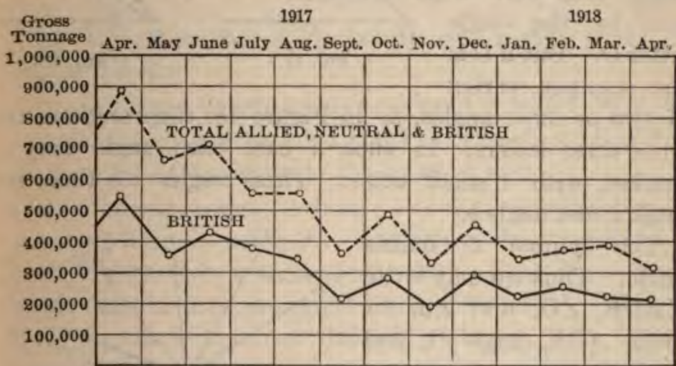
3. Given two lines x and y, x being greater than y, construct $3x - 2y$.

4. Show by construction that the difference between any two sides of a triangle is less than the third side.

5. A boy makes an error of $2' 8''$ in measuring the length of a lot which the surveyor says is $180'$ long. What per cent of error did he make?

6. Make a figure like Figure 14, page 127, using as radius $1\frac{1}{4}$ in. Compute the total length of all the arcs in the figure.

7. The graph below shows the decline in shipping losses from submarines from April 1917 to April 1918. Read the number of tons lost each month during that period.



A Year's Decline in Shipping Losses.

8. Suppose the earth to be a smooth sphere 25000 mi. in circumference. Suppose that a hoop whose inside circumference is 25000 mi. 1 ft. is placed around the earth at the equator. What is the distance between the inside of the hoop and the surface of the earth?

CHAPTER VIII

ANGLES

63. Meaning of angle. If two lines, OA and OB, drawn from the point O, an angle is formed.

The lines OA and OB are called the **sides of the angle**, the point O is called **the vertex**.

The angle in Figure 17 is read angle AOB, or angle B It should be noticed that the letter at the vertex is between the other two letters. This angle may also be read by the single letter O. But if O is the common vertex

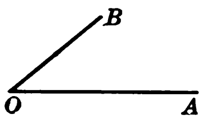


FIG. 17.

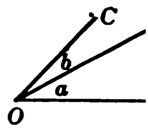


FIG. 18.

of two or more angles, as in Figure 18, each angle is with three letters. In such a case each angle may marked with a small letter. These angles are then angle a and angle b.

The symbol $\angle$ means angle. Thus we may write $\angle AOB$, $\angle O$, and $\angle a$, for angle AOB, angle O, and angle a.

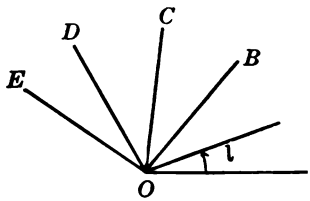


FIG. 19.

If in Figure 19 the line OA is turned about the point O, the point O itself remaining fixed, the line is *rotated about the point O.*

Suppose that a line l is placed upon the line OA in Fi so that one end of l falls at O. We may think of

$\angle AOB$ as being formed by rotating the line l about the point O in the direction of the arrow until it takes the position OB. If the line is rotated further we get $\angle AOC$, $\angle AOD$, and $\angle AOE$.

The *size* of an angle depends upon the amount of rotation required in making it. Thus, in Figure 19 $\angle AOC$ is greater than $\angle AOB$ since a greater amount of rotation is required in making $\angle AOC$.

The size of an angle is not changed by changing the lengths of its sides.

Two angles are **equal** if the same amount of rotation is required in making each of them. If two angles are equal, one can be placed upon the other so that their vertices coincide and the sides of one lie upon the sides of the other. This is the test most frequently used.

Exercise 68

1. With a ruler make an angle with vertex P and sides PQ and PR.

2. In Figure 19, what are the sides of $\angle AOD$? Of $\angle COD$?

3. Write the angles AOD, AOB, AOC, and AOE of Figure 19 in order of size, writing the smallest first.

4. On the blackboard draw a line OA. On this line place a ruler with one end at O. Now rotate the ruler about O so as to illustrate how an angle is made. Does the size of the angle depend upon the length of the ruler used? Upon what does the size of the angle depend?

5. If two angles are cut from paper, how can you test their equality?

6. With your ruler make such a figure as Figure 18. Cut it from paper, cutting along the lines OA and OC. Now fold along OB. Compare the sizes of angles AOB and BOC.

7. Point out angles about the schoolroom. Point out the sides and the vertex of each.

64. Kinds of angles. In Figure 20 the two lines AB and CD cutting each other at O form four angles, and the two lines are said to **intersect.**

Two angles such as $\angle AOC$ and $\angle COB$, which have the same vertex and a common side OC between them, are called **adjacent angles.** In Figure 19 angles BOC and COD are adjacent angles.

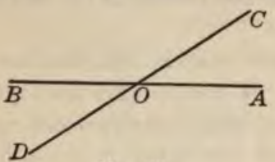


FIG. 20.

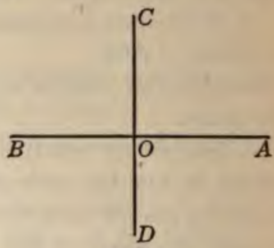


FIG. 21.

In Figure 20, $\angle AOC$ is smaller than its adjacent angle COB. Let the line CD be rotated about the point O until $\angle AOC$ equals $\angle COB$, as in Figure 21. Then the lines AB and CD are said to be **perpendicular** to each other and the angles formed are called **right angles.**

Definition. If one straight line meets another straight line making the adjacent angles equal, the lines are said to be perpendicular to each other and the angles formed are called right angles.

An *acute* angle is one that is less than a right angle.

An *obtuse* angle is one that is greater than one right angle and less than two right angles.

In Figure 20, $\angle AOC$ is acute and $\angle COB$ is obtuse.

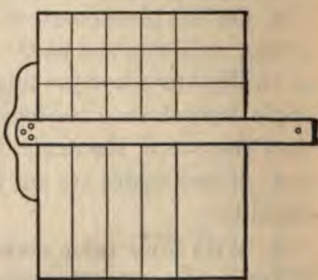


FIG. 22.

A T-square is often used by draftsmen in drawing right angles and parallel lines. See Figure 22.

Exercise 69

1. Draw an acute angle ; an obtuse angle.

2. What kind of angle is made by the edges of the cover of this book? By the edges of the floor of your schoolroom?

3. Open your book until the edges form an acute angle ; an obtuse angle.

4. Do the streets intersect at right angles in your city? Do you know of any streets that intersect at acute angles?

5. In Figure 19 read four pairs of adjacent angles and for each pair name the common side.

6. In Figure 20 is $\angle AOC$ larger or smaller than its adjacent angles?

7. In the adjoining figure are $\angle a$ and $\angle b$ adjacent? Why? Are $\angle a$ and $\angle c$ adjacent? Why? Name a pair of adjacent angles in this figure.

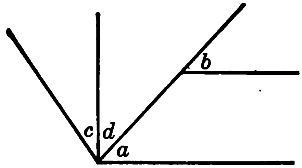


FIG. 23.

8. Take a piece of paper with a straight edge. Fold the paper so that one part of this straight edge lies along the other part. The line along which the paper is folded makes what kind of angle with the straight edge of the paper?

9. In Figure 24 what kind of angle is there between the lines pointing (*a*) east and north ; (*b*) east and southeast; (*c*) east and southwest; (*d*) east and northeast ; (*e*) northeast and northwest?

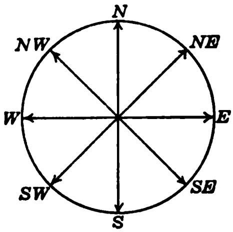


FIG. 24.

10. What kind of angles are formed by the edges of a cube?

11. *In the figure* of a pyramid on page 115, read an acute *angle ; a right* angle ; two pairs of equal angles.

65. The sum and the difference of angles. If the angles AOB and $DO'C$ are placed so as to be adjacent, then the angle AOC thus formed by their exterior sides is called the sum of angles AOB and $DO'C$. We may write

$$\angle AOB + \angle DO'C = \angle AOC.$$

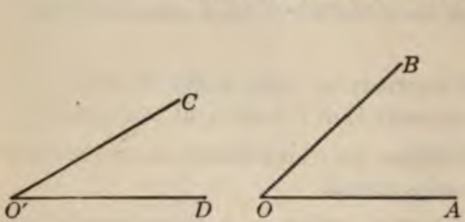


FIG. 25.

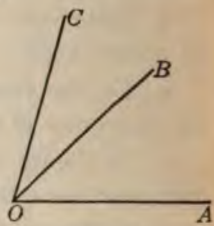


FIG. 26.

If the $\angle DO'C$ in Figure 25 is placed on the $\angle AOB$, so as to make Figure 27, so that O' falls on O, the side $O'D$ falls along OA, and the side $O'C$ falls within $\angle AOB$, then the $\angle COB$ thus formed is called the difference between $\angle AOB$ and $\angle DO'C$. We may write

$$\angle AOB - \angle DO'C = \angle COB.$$

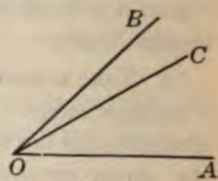


FIG. 27.

In Figures 26 and 27 the angles are said to be added and subtracted graphically.

66. Complementary and supplementary angles. Two angles whose sum is a right angle are said to be **complementary**, and each is said to be the **complement** of the other.

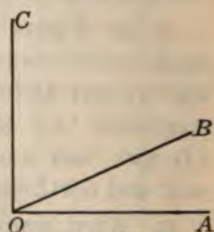


FIG. 28.

In Figure 28, $\angle AOB$ and $\angle BOC$ are complements of each other.

Two angles whose sum is two right angles are said to be **supplementary**, and each is said to be the **supplement** of the other.

In Figure 29, $\angle AOC$ and $\angle COB$ are supplements of each other.

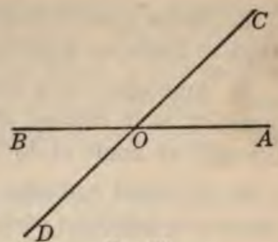


FIG. 29.

67. Measuring angles. In measuring lines we use a certain unit, as a foot, an inch, a mile. Name units used in measuring time; in measuring value; in measuring weight.

The unit usually used in measuring angles is $\frac{1}{90}$ of a right angle, which is called a **degree**. The symbol for degree is °. Thus we write 3 degrees as 3°.

This figure shows an angle of 1°.

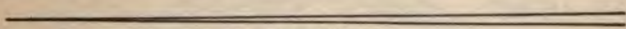


FIG. 30.

Exercise 70

1. How many degrees in a right angle? In four right angles?

2. How many degrees in one-half of a right angle?

3. How many degrees in the angles between the hands of a clock at three o'clock? At one o'clock? At six o'clock? At two o'clock? At four o'clock?

4. Through what angle does the minute hand of a clock rotate in 15 min.? In 30 min.? In 45 min.? In 60 min.? In 5 min.? In 10 min.? In 2 hr.? From twelve o'clock noon to twelve o'clock midnight?

5. What directions make angles of 45° with east? With south? With northeast?

6. Cut two angles from paper. Place them so as to show their sum and their difference.

7. Cut a triangle from paper. Mark the vertices A, B, and C. *Cut off* the corners and place them together *so as to show the sum* of the angles of the triangle.

8. Make a right angle by folding paper. Fold this angle so as to make an angle of 45°.

9. An engine on a turning table is headed east. In what direction is it headed after the table is turned to the right through an angle of 45°? 90°? 180°?

10. A wheel contains 16 spokes. What is the angle between two adjacent ones?

11. If $\angle A = 65°$, how many degrees in its complement? In its supplement?

12. How many degrees in $3\frac{1}{2}$ right angles? In $5\frac{1}{3}$ right angles? In a right angles? In x right angles?

13. How many right angles in an angle of 270°? Of 360°? Of $d°$? Of $m°$?

14. Draw an acute angle AOB. Draw a line OC so that $\angle BOC$ is the complement of $\angle AOB$. Draw a line OD so that $\angle BOD$ is the supplement of $\angle AOB$.

15. Find the supplement of an angle of 40°; $a°$; $x°$; $5k°$.

16. One angle contains $X°$ and another contains $Y°$, and $X+Y=90$. Find X if Y has the following values: 40°; 50°; 75°; 1°; 90°.

17. If two angles A and B are supplementary, then $\angle A + \angle B = 180°$. Find B if A has the following values: 15°; 90°; 75°; 0°; 179°; 180°.

18. May two angles be complementary but not adjacent?

19. Draw two obtuse angles which are adjacent. Are they supplementary? Why?

20. Draw two acute angles which are adjacent. Are they supplementary? Why?

21. May the complement of an acute angle be obtuse? Why?

22. May the supplement of an acute angle be obtuse? *Must it be so?* Why?

68. Using the protractor. The **protractor** is an instrument used in measuring and in drawing angles. The semicircle is divided into 180 equal parts, each of which is one degree. If these points of division are connected by straight lines with the center of the semicircle, angles of one degree are formed.

To measure $\angle AOB$ with the protractor, the center is placed at the vertex of the angle, and the radius OX is made to lie along the side OA of the angle, Figure 31. The number of angle degrees in $\angle AOB$ is the same as the number of arc degrees in arc XY. To find the number of degrees in $\angle AOB$ we read the number of degrees on the protractor from the

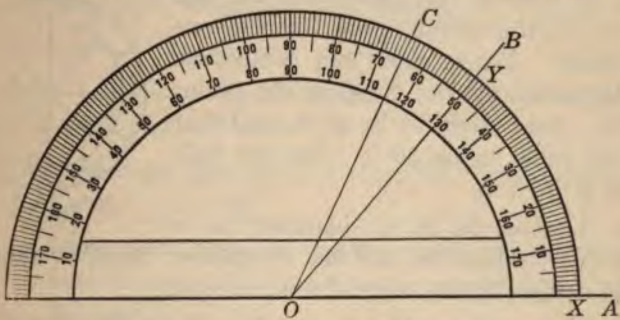


FIG. 31.

point X to the point Y. In Figure 31 $\angle AOB$ is seen to contain 50°.

The protractor may be used to construct an angle of a given size, for example an angle of 65°. To do this draw a line OA, Figure 31. Place the center of the protractor at O, and lay the side of the protractor, OX, along OA. Place a point C opposite 65°. Draw a line from O to C. Then $\angle AOC$ is the required angle.

The ruler, compasses, and protractor are drawing instruments *which the pupil* should learn to use with accuracy and *skill. Geometric* drawings should be accurate, neat, and legible.

Exercise 71

1. Measure the angles in Figure 18 with the protractor.

2. With the protractor make an angle of 90° ; of 30° ; of 57° ; of 120° ; of 180°.

3. Use the protractor to draw two lines that are perpendicular.

4. Make a figure such as Figure 24 on page 135, showing the directions. Make the angles as accurate as you can.

5. Construct a square with a side of 3 in. What kind of angles must be made at the corners?

6. In playing the game of " fox and geese " in the snow a figure like this one is made. Using ruler and compasses, make such a figure. Suppose the radius of the outer circle to be 40 ft. and that of the inner circle 30 ft. Use the scale of 20 ft. to 1 in.

FIG. 32.

7. A wheel 36 in. in diameter has a hub 4 in. in diameter and has 16 spokes. Make a diagram of this wheel on the scale of 9 in. to 1 in.

8. Make a figure like Figure 33. Make the sides of the square 2 in. Use the protractor in making the right angles. Use the side of the square as radius in making the arcs.

FIG. 33.

FIG. 34.

9. Make a copy of Figure 34. First find the point of intersection of the diagonals. Use the vertices of the square as centers and half of the diagonal as a radius in drawing the arcs.

69. Circular and angular measure. The following table is used in measuring both angles and arcs.

1 degree (°) = 60 minutes (′).
1′ = 60 seconds (″).

Exercise 72

1. Reduce 15° 24′ 8″ to seconds.

2. Reduce 60° 48′ 35″ to seconds.

3. Reduce 430′ to degrees and minutes.

4. Reduce 1235″ to minutes and seconds.

5. Reduce 125″ to minutes ; to degrees.

6. Over how many degrees does the end of the minute hand of a clock pass in 1 hr.?

7. Over what arc does the end of the hour hand pass in 1 hr.? In 1 min.? In 1 sec.?

8. In Italy clock faces are marked from 1 to 24. Over what arc does the end of the hour hand pass in 1 hr.? In 1 min.? In 1 sec.?

9. The latitude of New York is 40° 42′ N., of London, 51° 30′ N. Find the difference in their latitudes.

10. The latitude of New Orleans is 29° 51′ 45″ N., of Buenos Aires, 34° 26′ 21″ S. Find the difference in their latitudes.

11. Find the difference in latitude between Chicago, latitude 41° 53′ 6″ N., and Constantinople, latitude 41° 0′ 16″ N.

12. A ship sails south at the average rate of 1° 42′ 37″ a day for 8 days. Find the total distance sailed south measured in degrees, minutes, and seconds.

Longitude and Time

70. How longitude is measured. It has been learned in geography that longitude is measured east and west from a principal meridian. The longitude of places in the United States is measured from the principal meridian running through Greenwich, England.

Below are given the longitudes measured from Greenwich, of eleven cities, correct to 1′.

Boston . . .	71° 4′ W.	Manila, Philippines .	120° 58′ E.
Chicago . . .	87° 38′ W.	Melbourne	144° 58′ E.
Constantinople	28° 59′ E.	New York	74° 0′ W.
Denver . . .	104° 59′ W.	San Francisco . . .	122° 25′ W.
London . . .	0° 6′ W.	Tokio	139° 40′ E.
		Washington . . .	77° 0′ W.

Exercise 73

1. The captain of a ship finds that he is in longitude 56° W. How many degrees east of New York is he?

2. A ship is in longitude 8° W. when it gets a wireless message from a ship in longitude 12° E. What is the difference in longitude of the two ships?

3. Find the difference in longitude of two places A and B (*a*) if A is in longitude 30° W. and B 73° W. (*b*) If A is in longitude 40° E. and B 147° E. (*c*) If A is in longitude 56° W. and B 98° E. (*d*) If A is in longitude 145° E. and B 32° W.

4. A ship in longitude 8° W. sails west 24°. What is its longitude then?

5. A ship in longitude 15° E. sails west 84°. In what longitude is it then?

6. If a ship sails east 47° from longitude 150° E. what is its new longitude? Show this on a globe.

7. What is the difference in longitude between 178° W. and 178° E.?

8. A captain finds that he is in longitude 150° W. He knows the longitude of Melbourne. How can he find out how many degrees of longitude he must pass over to reach Melbourne?

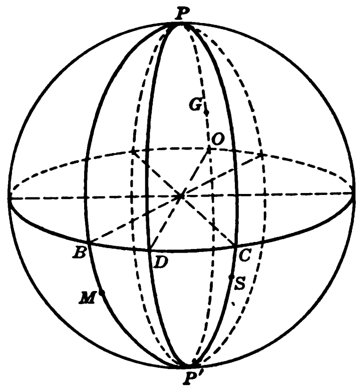


FIG. 35.

Hint. In Figure 35, suppose that the ship is at S, that Melbourne is at M, and that PGP' is the meridian of Greenwich. How many degrees in the arc OC? How many degrees in the arc OB? How many in the arc COB? How many in the shorter arc BDC? What is the difference in longitude of Melbourne and the ship?

9. State how to find the difference in longitude of two places on the same side of the principal meridian.

10. State how to find the difference in longitude of two places on opposite sides of the principal meridian, when the sum of their longitudes is less than 180°.

11. State how to find the difference in longitude of two places on opposite sides of the principal meridian, when the sum of their longitudes is greater than 180°.

12. Find the difference in longitude between Chicago and New York.

13. Find the difference in longitude between San Francisco and Constantinople.

14. Find the difference in longitude between Boston and Manila.

15. Find the difference in longitude between Washington and Tokio.

By using any fixed point to represent the sun, and holding a globe at some distance from it, the pupil can see the meaning of noon, twilight, and midnight. By rotating the globe from west to east he will see that the sun time of a place west of a given meridian is earlier than that of the meridian, while the time at a place east of the meridian is later than that of the meridian.

If told that it is noon at A we know that the meridian of A is directly under the sun. If B is 15° west of A, then B must turn through 15° for its meridian to be directly under the sun. This turning through 15° will require one hour. Therefore, it is one hour before noon at B when it is noon at A. In the same way it may be seen that the meridian of C, 15° east of A, has passed directly under the sun one hour before that of A. Then, if it is noon at A it is one hour past noon at C.

The following table shows what changes in longitude correspond to given changes in time.

In 24 hours the earth turns through 360°.

In 1 hour the earth turns through $\frac{1}{24}$ of 360°, which equals 15°.

In 1 minute the earth turns through $\frac{1}{60}$ of 15°, which equals $\frac{1}{4}$°, or 15′.

In 1 second the earth turns through $\frac{1}{60}$ of 15′, which equals $\frac{1}{4}$′, or 15″.

The following table shows what changes in time correspond to given changes in longitude.

The earth rotates through :

360° in 24 hours.

1° in $\frac{1}{360}$ of 24 hours, which equals $\frac{1}{15}$ of an hour, or 4 minutes.

1′ in $\frac{1}{60}$ of 4 minutes, which equals $\frac{1}{15}$ of a minute, or *4 seconds.*

1″ in $\frac{1}{60}$ *of* 4 seconds, which equals $\frac{1}{15}$ of a second.

Exercise 74

1. Learn the tables given above so that you can give them accurately and readily.

2. A point on the earth's surface rotates through what arc in 1 day? In 1 hr.? In 1 min.? In 1 sec.?

3. Through how many degrees does the point where you are located rotate from noon Monday to noon Tuesday? From noon to midnight? From 11 A.M. to noon? From 10 A.M. to noon? From 9 A.M. to noon? From 6 A.M. to noon?

4. If a certain meridian is under the sun, that is, if it is noon along that meridian, has a meridian 15° west of that one passed under the sun on the same day? How long before it will do so? How long since it was noon on a meridian 30° east?

5. If it is noon by the sun at Chicago, is it before or after noon at Denver? At New York? At Boston? At places east of Chicago? At places west?

6. What time is it where you are now? Is it earlier or later at places east? At places west?

7. It is noon at A. Through how many degrees must the earth rotate before it is one o'clock at A? Two o'clock? Six o'clock?

8. If it is noon where you are, what time is it 15° west? 30° west? 45° west? 15° east? 45° east? 60° east?

9. What is the difference in time between two places if the difference in longitude is 1°? 1′? 1″?

10. When it is 9 A.M. in New York, what time is it in Denver?

SOLUTION. Denver is 30° 59′ west of New York.
For the earth to rotate through 1° requires 4 min.
To rotate through 30° requires 30×4 min. = 120 min.
= 2 hr.

$$\begin{array}{rr} 104 & 59 \\ 74 & 0 \\ \hline 30 & 59 \end{array}$$

To rotate through 1′ requires 4 sec.

To rotate through 59′ requires 59×4 sec. $= 236$ sec.
$= 3$ min. 56 sec.

Since Denver is west of New York the time is 2 hr. 3 min. 56 sec. earlier in Denver than in New York.

2 hr. 3 min. 56 sec. earlier than 9 A.M. is 6 : 56 : 4 A.M.

$$\begin{array}{rrr} 9 & 0 & 0 \\ 2 & 3 & 56 \\ \hline 6 & 56 & 4 \end{array}$$

11. When it is noon in San Francisco what time is it in London?

12. One ship is in longitude 42° 36′ 10″ and another is in longitude 55° 12′ 10″. Find the difference in longitude and the difference in time.

13. The time in New York is 4 min. 10 sec. earlier than in New Haven. Find the longitude of New Haven.

14. When it is noon at Chicago it is 1 : 12 P.M. at a certain other place. Find the longitude of this place.

15. If a wireless signal is sent at 4 P.M. from a ship in longitude 58° 20′, at what time is it received in Boston, allowing no time for transmission?

16. The time in A is 4 hr. 25 min. 6 sec. later than in B. Find their difference in longitude and tell which is the farther west.

17. Find the time in Melbourne when it is 8 : 30 P.M. in Chicago.

18. Find the time in Tokio when it is 10 : 20 A.M. in San Francisco.

19. Making no allowance for time of transmission, a message started from New York to Manila at 11 : 35 A.M. would reach Manila at what time?

20. Allowing two hours for transmission, a cablegram *started from* London to Washington at 4 : 25 P.M. would *reach Washington* at what time?

71. Standard time. The time that has been used in the above problems is called **local time**. The continuous change of local time in going east or west from a given point caused

FIG. 36. — Map showing standard time belts of the United States.

great inconvenience in railroad time schedules. In 1883 the railroads of the United States adopted a system of time called **standard time**, which is much simpler for purposes of transportation.

In making this system of time the United States was divided into **standard time belts**, Eastern, Central, Mountain, and Pacific. The time in the Eastern belt is the same as the local time of the 75th meridian ; for example, when it is noon, local time, on the 75th meridian, it is noon, standard time, throughout the Eastern belt. The Central, Mountain, and Pacific belts take the local time of the 90th, the 105th, and the 120th meridians respectively. The lines dividing the time belts do not follow meridians but run through important railroad centers (Figure 36).

In nearly all places in the United States the time kept by the clocks is standard time.

Exercise 75

1. When it is noon, standard time, in New York what is the standard time in St. Louis? In Boston? In Denver? In Chicago? In San Francisco?

2. What standard time do you use?

3. At what hour, standard time, does your arithmetic class meet? What is the standard time then in Portland, Maine? In Florida? In New Orleans? In Utah?

4. In going from New York to Seattle what changes in time would you make?

5. Find from a map the longitude of the place in which you live, and find the difference between its local time and its standard time.

6. When it is noon on the 90th meridian what is the local or sun time on the 85th meridian? What is the standard time on that meridian? What is the difference between local time and standard time on the 85th meridian? Which is faster?

7. What is the difference between the local time and the standard time on the 100th meridian?

8. When the clock says 10 : 30 A.M., standard time, on the 82d meridian what is the sun time?

9. The standard time of a place in the Mountain belt is 25 minutes faster than its local time. Find its longitude.

10. What is the local time in Chicago when it is 9 A.M., standard time?

11. The standard time in London is 5 hours later than that of New York and that of Rome is 6 hours later than that of New York. By how much do these differ from the differences in local time? The longitude of Rome is 12° 29′ E.

12. When it is 11 A.M. by the standard time of San Francisco, what is the standard time of London?

Exercise 76. Review

1. The following average heights for different ages for American boys and girls were found by measuring 45,151 boys and 43,298 girls in different American cities. Represent these heights graphically.

Age in years	$5\frac{1}{2}$	$6\frac{1}{2}$	$7\frac{1}{2}$	$8\frac{1}{2}$	$9\frac{1}{2}$	$10\frac{1}{2}$	$11\frac{1}{2}$
Av. height in inches, boys	41.7	43.9	46.0	48.8	50.0	51.9	53.6
Av. height in inches, girls	41.3	43.3	45.7	47.7	49.7	51.7	53.8

Age in years	$12\frac{1}{2}$	$13\frac{1}{2}$	$14\frac{1}{2}$	$15\frac{1}{2}$	$16\frac{1}{2}$	$17\frac{1}{2}$
Av. height in inches, boys	55.4	57.5	60.0	62.9	64.9	66.5
Av. height in inches, girls	56.1	58.5	60.4	61.6	62.2	62.7

2. Define circle, radius, diameter, arc, degree of arc.

3. Define right angle, acute angle, obtuse angle.

4. Find the complement of an angle of $20°$; $12° 30'$; $0°$; $90°$; $m°$; $89° 59' 59''$.

5. Find the supplement of an angle of $65°$; $90°$; $179°$; $58°$; $180°$; $x°$; $0°$; $1''$; $1'$.

6. Define a plane surface. Name three solids whose faces are plane surfaces.

7. What does latitude mean? What does longitude mean? How many degrees of latitude from the equator to the north pole? From the north pole to the south pole? What is the greatest latitude that a place may have? What is the greatest longitude?

8. Through how many degrees, minutes, and seconds of arc does a point on the earth's surface turn in one day? In one hour? In one minute? In one second?

9. The difference in longitude of two ships is $12° 46'$. What is their difference in time?

10. If the radius of a circle is doubled, is the diameter doubled? Is the circumference doubled? Give examples.

11. A horse is tethered by a rope 100 ft. long to the corner of a barn 40 ft. by 50 ft. Make a figure to scale to show the whole area over which the horse can graze.

12. By the census of 1910 the foreign-born population of the United States came as follows from these countries:

Germany	2,501,333
Russia	1,732,462
Austria	1,670,582
Ireland	1,352,251
Italy	1,343,125
Canada	1,209,717
Great Britain	1,221,283
All others	2,485,133

FIG. 37.

This distribution is shown in the graph, Figure 37. In this graph 1° should represent how many people? With the protractor measure the angle of the part of the circle given to each country. What is the sum of these angles? Does the figure give the correct number of degrees to Ireland?

13. In an arithmetic class of 40 pupils the grades were as follows: 5 were marked excellent; 12, good; 13, fair; 7, passed; 3, failed. Make a graph as in the last exercise to show the number having each grade.

HINT. One pupil corresponds to how many degrees on the graph?

14. The census of the United States for 1910 showed the following population:

Native white	68,386,412	Negro	9,827,763
Foreign-born white	13,345,545	Other colored races	412,546

Make a graph such as in Figure 37 to represent this division of the population.

HINT. First find how many people correspond to 1°.

15. *Find what per cent of the total population is found in each class of the* [illegible]ion given in exercise 18.

CHAPTER IX

TRIANGLES. CONSTRUCTIONS WITH RULER AND COMPASSES

TRIANGLES

72. Definitions. A portion of a plane bounded by three straight lines is called a **triangle**.

The three bounding lines are called the **sides** of the triangle. The vertices of the angles are called the **vertices** of the triangle.

A triangle is named by reading the letters at its vertices in any order. The symbol $\triangle$ is used to represent the word triangle. Thus, in Figure 38 we have $\triangle ABC$ which is read triangle ABC.

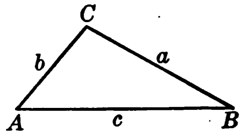


FIG. 38.

The **perimeter** of a figure is the distance around it.

It is often convenient to letter a triangle as in Figure 38, so that the side a is opposite the angle A, the side b is opposite the angle B, and the side c is opposite the angle C.

73. Classification of triangles. A triangle is classified as to the kinds of angles it has.

An **acute triangle** is one all of whose angles are acute.

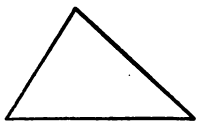


FIG. 39.—Acute.

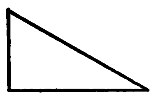


FIG. 40.—Right.

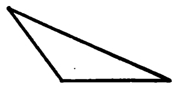


FIG. 41.—Obtuse.

An **obtuse triangle** is one which has one obtuse angle.

A ***right triangle*** is one which has one right angle.

An **equiangular triangle** is one all of whose angles are equal.

A triangle is classified as to the relative length of its sides.

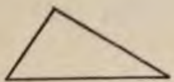


FIG. 42. — Scalene.

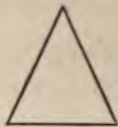


FIG. 43. — Isosceles.

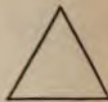


FIG. 44. — Equilateral.

A **scalene triangle** has no two sides equal.

An **isosceles triangle** has two equal sides.

An **equilateral triangle** has its three sides equal.

74. Base and altitude of a triangle. A line from a vertex of a triangle perpendicular to the opposite side is called an **altitude** of the triangle.

The side to which the altitude is perpendicular is called the **base** of the triangle.

In Figure 45, CM is an altitude and AB is the base.

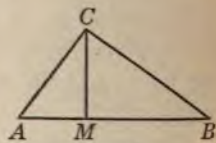


FIG. 45.

Exercise 77

1. The sides of a triangle are 5 in., 7 in., and 10 in. Find its perimeter.

2. The perimeter of an equilateral triangle is 36 inches. Find the length of one side.

3. One of the two equal sides of an isosceles triangle is x. The third side is 17 in. The perimeter is 63 in. Find x.

4. This figure shows the *three altitudes* of the obtuse *triangle* ABC. Read them.

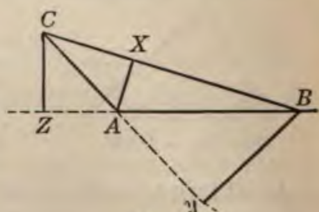


FIG. 46.

Using a ruler draw as accurately as you can the three altitudes of another obtuse triangle.

5. Draw an acute triangle and draw its three altitudes.

6. Draw a right triangle and its three altitudes.

7. Is an equilateral triangle also isosceles?

8. The perimeter of an equilateral triangle is a feet. Find the length of one side.

9. How many triangular faces has the pyramid on page 115? What kind of triangles are these faces?

Constructing Figures

75. The use of ruler and compasses. We are frequently required to construct certain figures to meet certain requirements. We may need to draw a square having its side just 2 inches long, or to construct a triangle having one of its angles just 30 degrees. In making these and other constructions accurately the ruler is used only to draw straight lines and the compasses to draw circles and mark off segments equal to given segments.

76. To construct a triangle with three given sides.

Let the given sides be a, b, and c.

Construction. Draw a straight line AX and on this line mark off the segment AB equal to c. With A as a center

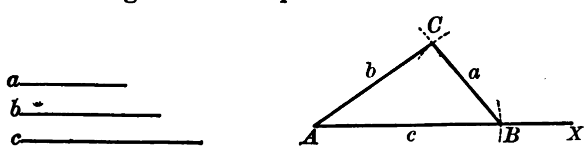


Fig. 47.

and radius b construct an arc. With B as center and radius a construct an arc cutting the first arc at a point C. Draw the lines AC and BC. The triangle ABC is the triangle *required.*

77. At a given point in a given line to construct an angle equal to a given angle. Given the angle O and the point P on the line MN, to construct at P an angle equal to the angle O.

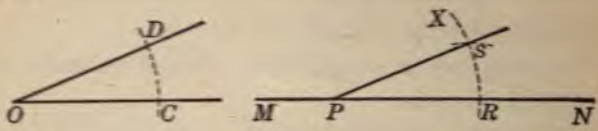


FIG. 48.

CONSTRUCTION. With O as center and any convenient radius draw an arc cutting the sides of $\angle O$ at points C and D.

With P as center and radius OC draw an arc RX cutting the line PN at R.

With R as center and radius CD draw an arc cutting the arc RX at S. Draw PS.

Then the angle RPS is the desired angle.

Exercise 78

1. Construct a triangle with sides $1\frac{1}{2}$ in., 2 in., and $2\frac{1}{2}$ in.

2. Construct an equilateral triangle with sides 1 in.

3. Construct an isosceles triangle with base $1\frac{1}{4}$ in., and the equal sides $1\frac{7}{8}$ in. each.

4. Make an angle. Construct an angle equal to it. Test their equality by measuring the angles with the protractor.

5. Make an obtuse angle. Construct an angle equal to it.

6. A triangular field has sides 10 rods, 14 rods, and 17 rods. Make a drawing of this field on the scale of 8 rods to the inch.

7. *Construct* a triangle with sides twice as long as those *of Figure 47.*

8. Construct an angle equal to the sum of two given angles as in Figure 49.

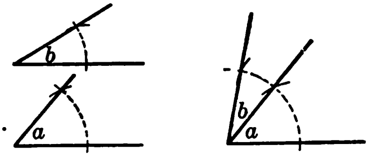


FIG. 49.

9. Make an acute angle and an obtuse angle. Construct their sum.

10. Make a triangle. Construct an angle equal to the sum of the three angles of this triangle.

11. Draw two angles. Construct an angle equal to their difference.

12. Measure the angles of the triangle in exercise 2. How do the angles compare in size? Do the same for the triangle in exercise 3 and compare the angles.

78. To construct a triangle when given two sides and their included angle. In $\triangle ABC$, Figure 50, the $\angle A$ is said to be **included** by the sides AB and AC, and the side AB is said to be **included** by the angles A and B.

We wish to construct a triangle having the sides c and b, and the $\angle O$ included between them.

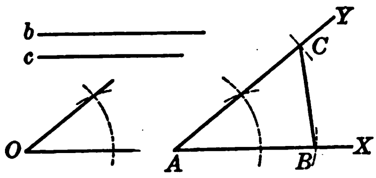


FIG. 50.

CONSTRUCTION. Draw a line AX. At A construct $\angle XAY$ equal to $\angle O$.

On AX mark off AB equal to line c, and on AY mark off AC equal to line b. Join B and C.

Then $\triangle$ ABC *is* the triangle required.

79. To construct a triangle when given two angles and their included side. Given $\angle O$ and $\angle Q$ and their included side a, we wish to construct a triangle having these parts.

CONSTRUCTION. Draw a line CB equal to a.

At C construct an angle equal to $\angle O$, and at B construct an angle equal to $\angle Q$.

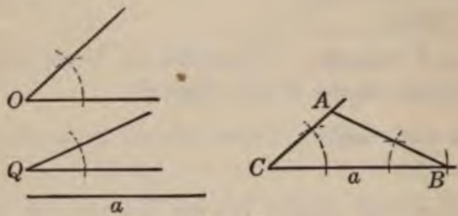


FIG. 51.

Prolong the sides of these angles until they meet at a point A.

Then $\triangle\ ABC$ is the desired triangle.

Exercise 79

1. Construct a triangle given two sides and the included angle. Let the given angle be obtuse.

2. Construct a triangle given two angles and the included side.

3. A railroad cuts off from a farm a triangular field with side $a=20$ rods, side $b=15$ rods, and $\angle C=90°$. Draw such a triangle on the scale of 10 rods to 1 inch. Use the protractor or T-square in making the angle of 90°.

4. Construct an equilateral triangle. Measure the angles. How many degrees in each angle? How do the angles compare in size?

5. Construct an isosceles triangle. Measure its angles. What is true of the angles opposite the equal sides?

6. Construct a triangle with sides 1 in., $1\frac{1}{2}$ in., and 2 in. Measure the angles. Which angle is greatest? Which side is it opposite? Which angle is smallest? Which side is it opposite? Measure the angles of the triangle of example 4 and answer the same questions. Which of the principles stated below is illustrated here?

7. Find the sum of the angles of each of the triangles of examples 4, 5, and 6. If you could measure the angles of a triangle exactly, by how much do you think their sum would differ from 2 right angles?

8. Draw a triangle. Construct with ruler and compasses an angle equal to the sum of the angles of this triangle. Measure with the protractor this angle which you have constructed. How many degrees does it contain?

9. Can you make a triangle with the sides 2 in., 1 in., and 3 in.? Try it.

10. Can you make a triangle with sides 1 in., $\frac{3}{4}$ in., and 2 in.? Try it.

11. Make the Gothic Arch, Figure 52.

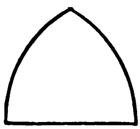


FIG. 52.

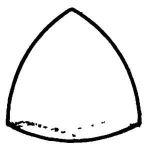


FIG. 53.

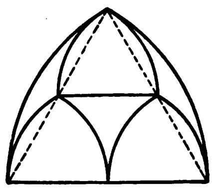


FIG. 54.

12. Make Figure 53.

13. Make Figure 54.

14. Use the protractor to make a five-pointed star. How many degrees in the arc AB? With the protractor *construct an angle* of that number of *degrees at the* center of the circle.

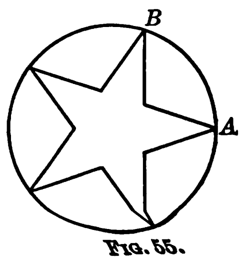


FIG. 55.

Principles Concerning Triangles

80. Facts to be learned. The following important principles concerning triangles have been illustrated in the preceding exercises. They should be memorized. In what exercise is each illustrated?

I. The sum of the angles of a triangle equals 180°.

II. The angles of an equilateral triangle are equal.

III. In an isosceles triangle the angles opposite the equal sides are equal.

IV. In any triangle if two sides are unequal, the angles opposite those sides are unequal and the angle opposite the greater side is the greater.

V. The sum of two sides of a triangle is greater than the third side.

Exercise 80

1. Can a triangle have two right angles? Why? Two obtuse angles? Why? Two acute angles? What is the least number of acute angles that a triangle can have? Why?

2. In a right triangle how large may one of the acute angles be?

3. In a triangle ABC, $a=16$ ft., $b=20$ ft., and $c=10$ ft. Which is the largest angle? Which is the smallest?

4. In a triangle ABC, $\angle A=48°$, and $\angle B=65°$. How many degrees in $\angle C$? Which side is the longest? Which side is the shortest?

5. One angle of a right triangle contains 72°. How many degrees in the smallest angle?

6. One of the equal angles of an isosceles triangle is 50°. How many degrees in each of the other angles?

7. Can the angles of a triangle be 101°, 38°, and 35° respectively?

8. One angle of a triangle is $48° 20' 10''$ and another is $76° 42' 56''$. Find the third angle.

9. One angle of a triangle is $96° 42' 43''$ and another is $83° 17' 16''$. Find the third angle.

10. In $\triangle ABC$, $\angle A = 34°$, and $\angle B = 62°$, and side $a = \frac{5}{8}$ in. Construct the triangle. Which side is the longest? Which is the shortest?

11. Can you construct a triangle ABC with $\angle A = 127°$, $\angle B = 64°$, and $AB = 2$ in.? If you cannot, give a reason.

81. The perpendicular bisector of a line.

1. Take a point P. How many different straight lines can be drawn to pass through P?

2. Take two points M and N. How many different straight lines can be drawn to pass through both M and N?

3. If you wish to draw a certain line and must first find some points on it, how many points must you find in order to be able to draw it?

The answers to these questions make clear the following

Principle. *Through two points only one straight line can be drawn.*

This principle may be stated in the form :

Two points determine a straight line. This means that one straight line and only one can be drawn through the two points.

If a line is divided into two equal parts, it is said to be **bisected.**

82. To construct the perpendicular bisector of a given line segment. We wish to construct the perpendicular bisector of the segment AB, that is, to construct a line which is perpendicular to AB, and which bisects AB.

How *many points* on the perpendicular bisector must we *know in order* to be able to draw it?

If P is a point on the perpendicular bisector, how does the distance from P to A compare with the distance from P to B? With the compasses find such a point. Now find another such point, Q. If you then join the points P and Q, you will have the perpendicular bisector of AB.

The construction may be made as follows:

With A and B as centers and the same radius construct arcs meeting at P and Q. Draw the line PQ, which intersects AB in a point M. Then PQ is the perpendicular bisector and M is the mid-point of AB.

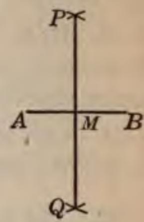


FIG. 56.

Exercise 81

1. Draw a line segment and bisect it.
2. With ruler and compasses make the following figures.

FIG. 57.

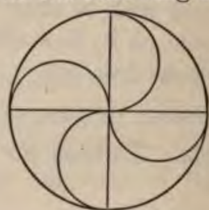


FIG. 58.

3. With ruler and compasses make the following figures.

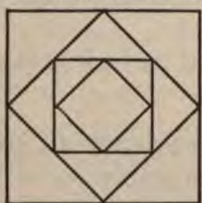


FIG. 59.

FIG. 60.

4. In a given circle draw a chord. Draw the perpendicular bisector of the chord. Does the perpendicular bisector

of the chord pass through the center of the circle? Draw another chord and its perpendicular bisector. Where do the two perpendicular bisectors meet?

5. If you do not know where the center of a given circle is, how can you find it?

6. If you were given part of the rim of a wheel, as in Figure 61, how could you find the length of its radius?

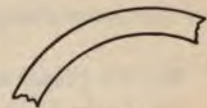


FIG. 61.

Principle. *The perpendicular bisector of a chord passes through the center of the circle.*

83. To erect a perpendicular to a line at a given point on the line.

We wish to erect a perpendicular to the line AB at the point P.

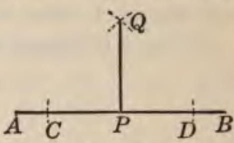


FIG. 62.

With P as center construct an arc cutting AB at C and D.

With C as center and radius greater than one-half CD, construct an arc.

With D as center and the same radius construct an arc cutting the last arc at Q. Draw PQ.

Then PQ is perpendicular to AB at P.

84. To erect a perpendicular to a given line from a given external point.

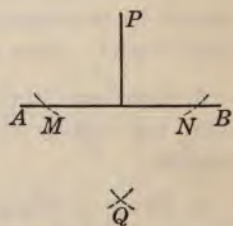


FIG. 63.

It is required to erect a perpendicular to the line AB from the point P.

With P as center construct an arc cutting AB at M and N.

With M and N as centers and equal radii construct arcs cutting each other at Q. Draw the line PQ.

Then PQ is the perpendicular to the line AB from the point P.

85. The distance from a point to a line. If you were in a field and wished to take the shortest path to a straight road AB, you would follow the perpendicular from the point to the line AB.

By the **distance** from a point to a line we mean the length of the perpendicular from the point to the line.

86. To bisect a given angle.

An angle is bisected by a line which divides it into two equal angles.

It is required to bisect $\angle ABC$.

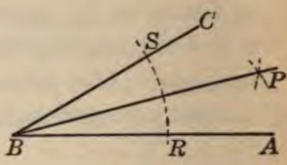


FIG. 64.

With B as center draw an arc cutting AB at R and BC at S.

With R and S as centers and the same radius draw arcs cutting each other at P. Draw the line PB.

Then PB bisects the $\angle ABC$.

Exercise 82

1. Along what line would you measure the distance from a point on the blackboard to the upper edge of the blackboard?

2. A straight railroad runs through a town. What is meant by the distance from a point in the town to the railroad track? Draw a figure to illustrate your answer.

3. Construct the perpendicular bisector of a given line. Test the accuracy of your construction by measuring the angles.

4. Draw a triangle ABC. Construct the perpendicular bisector of each side. If the construction is accurate, these bisectors will meet in a point. Call this point O. Using *the compasses,* compare the distances OA, OB, and OC. *Can you now draw* a circle passing through A, B, and C?

5. Take three points not in a straight line. Construct a circle passing through these three points.

HINT. Join the three points, thus forming a triangle, and use the method of the preceding exercise.

6. Construct an angle of 45°.

HINT. Construct a right angle and bisect it.

7. What angle, if bisected, will give an angle of 22° 30′? Construct an angle of 22° 30′.

8. Draw a triangle and construct the bisector of each of its angles. If the constructions are accurate, these bisectors will meet in a point.

9. Erect a perpendicular to the line AB from the external point C.

10. Define the altitude of a triangle.

11. Draw an acute triangle. Construct its three altitudes. If the constructions are accurate, the three altitudes will meet in a point.

12. Draw an obtuse triangle, and construct its three altitudes. The same test applies as in the preceding exercise.

13. Draw a right triangle. Construct its three altitudes. Do they meet in a point? Where?

14. Construct these eight-pointed stars.

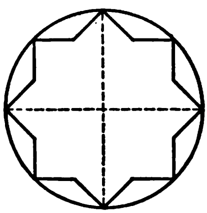


FIG. 65.

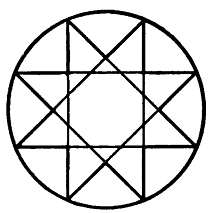


FIG. 66.

HINT. Draw two perpendicular diameters and bisect the angles between them to obtain the vertices.

15. Construct an equilateral arch.

HINT. Draw an equilateral triangle and bisect its sides.

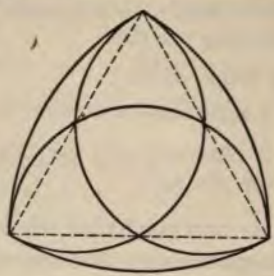


FIG. 67.

This picture from Durham Cathedral, England, shows the use of geometric forms in architectural decoration.

CHAPTER X

PARALLEL LINES, QUADRILATERALS, AND REGULAR POLYGONS

87. Parallel lines. On page 125 the pupil learned what a plane surface is. Give some examples of plane surfaces. Give examples in the schoolroom of two lines in the same plane. Give examples of lines not in the same plane. Can you give examples of two lines in the same plane that will not meet however far they are produced? (Produced means extended.)

Two lines in the same plane that will not meet however far they are produced are called **parallel lines.**

The opposite edges of a ruler, the top and bottom of the blackboard, may be taken as examples of parallel lines.

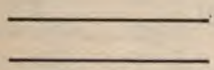


FIG. 68.

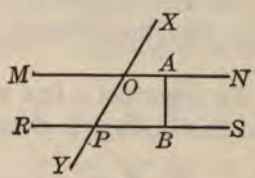


FIG. 69.

A line that intersects two or more lines is called a **transversal** of those lines.

In Figure 69 XY is a transversal of MN and RS.

The **distance between two parallel lines** is the length of the perpendicular between them. In Figure 69 AB is the *distance between* MN and RS.

Exercise 83

1. Point out examples of parallel lines about the schoolroom.

2. Give examples of lines in the same plane that are not parallel.

3. Give an example of two lines not in the same plane.

4. Use the opposite edges of a ruler for making lines MN and RS and make a figure similar to Figure 69, omitting the line AB. With the protractor measure the angles made by the transversal. Copy and complete the following table.

ANGLE	NOX	XOM	MOP	PON	SPO	OPR	RPY	YPS
Number of degrees								

5. If a line MN is drawn through a point P parallel to AB, Figure 70, $\angle NPX$ must be made equal to what angle? Can you construct through P a line parallel to AB?

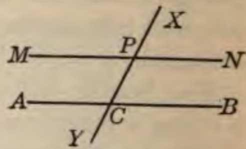


FIG. 70.

6. Construct a parallel to a line AB through a point P.

HINT. First draw through P a line XY cutting AB at a point C.

88. To construct a line parallel to a given line through a given external point.

It is required to construct a line parallel to the line AB through the point P.

Through the point P construct the line XY cutting AB at the point C.

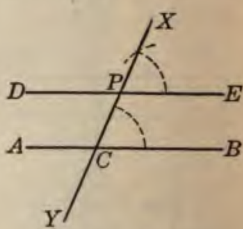


FIG. 71.

Through P draw a line DE making $\angle EPX$ equal to $\angle BCX$.

The line DE is the required line.

Exercise 84

1. Using the protractor, make an angle of 60°. Construct an angle equal to it, using ruler and compasses. Test the accuracy of your construction by using the protractor.

2. Through a given point Q draw a line parallel to a given line MN.

3. Construct two parallel lines AB and CD. Construct MN and PQ perpendicular to AB. Are MN and PQ also perpendicular to CD? Are MN and PQ equal?

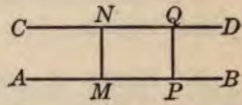


FIG. 72.

4. Are two parallel lines the same distance apart at all points?

5. A contractor wishes to lay a concrete walk 6 ft. from the wall of a building and parallel to it. State two ways of locating the edge of the walk so that it will be parallel to the wall.

6. Construct parallel lines 1 in. apart.

7. Construct the design given in Figure 73. Notice the parallel lines. Upon what kind of triangles is this design based?


FIG. 73.

8. Construct the design given in Figure 74. Notice the size of the angles, and the parallel lines.


FIG. 74.

9. Construct the linoleum pattern of Figure 75. It is based upon how many sets of parallel lines?

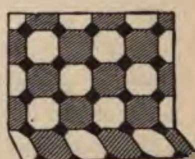


FIG. 75.

89. Quadrilaterals. A portion of a plane bounded *by four* lines is called a **quadri-*lateral*.**

A quadrilateral is classified according to the number of its parallel sides.

A quadrilateral with no parallel sides is called a **trapezium.**

A quadrilateral with only one pair of parallel sides is called a **trapezoid.**

A quadrilateral with two pairs of parallel sides is called a **parallelogram.**

90. Kinds of parallelograms. A parallelogram whose sides are all equal is called a **rhombus.**

A parallelogram whose angles are all right angles is called a **rectangle.**

A rectangle whose sides are all equal is called a **square.**

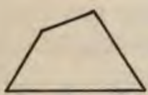


FIG. 76.—Trapezium.

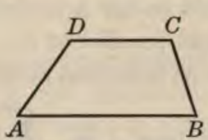


FIG. 77.—Trapezoid.

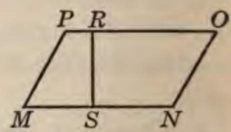


FIG. 78.—Parallelogram.

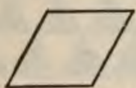


FIG. 79.—Rhombus.

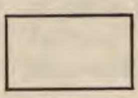


FIG. 80.—Rectangular.

FIG. 81.—Square.

91. Base. Altitude. Diagonal. The side upon which a figure is supposed to rest is called the **base.**

The parallelogram and the trapezoid are said to have an **upper base** and a **lower base.** MN is the upper base and OP the lower base of the parallelogram $MNOP$.

The perpendicular distance between the bases of a parallelogram or of a trapezoid is called the **altitude.** RS is the altitude of the parallelogram $MNOP$.

*The **diagonal** of a quadrilateral is a line joining two opposite vertices.*

92. To construct a parallelogram. It is required to construct a parallelogram, given two adjacent sides, x and y, and their included angle O.

First construct $\angle A$ equal to the given angle O.

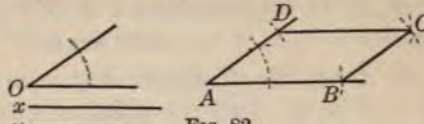


FIG. 82.

On the sides of $\angle A$ mark off AB and AD equal to x and y respectively.

With B as a center and radius y construct an arc. With D as center and radius x construct an arc cutting the arc last constructed at C.

Draw lines BC and DC. Then $ABCD$ is the required parallelogram.

Exercise 85

1. With ruler and compasses construct a rectangle with base 2 in. and altitude 1 in.

2. Construct a square with side $1\frac{1}{8}$ in.

3. How many degrees in the sum of the angles of a rectangle? Of a square?

4. Construct a parallelogram $MNOP$, given two adjacent sides MN and NP, and their included angle M.

5. With a protractor measure the four angles of the parallelogram $MNOP$. Find the number of degrees in the sum of the angles M and N; in the sum of angles N and O; in the sum of angles O and P; in the sum of angles P and M.

6. Construct a parallelogram given two adjacent sides and their included angle, which is obtuse. Measure the angles with a protractor. Compare the opposite angles. Find the sum of each pair of adjacent angles.

7. *State any fact you have discovered about the opposite angles of a parallelogram;* about the adjacent angles.

8. How many degrees are there in the sum of the angles of parallelogram $MNOP$?

9. If, in a parallelogram $MNOP$, $\angle M = 60°$, how many degrees in each of the other angles?

10. Draw the diagonals of the parallelogram constructed in exercise 4. Call the point of intersection A. Measure and compare the lengths of MA and AO, and of NA and AP. What conclusion can you draw from these measurements? Test this conclusion in other parallelograms.

11. Construct a rectangle and draw its diagonals. Measure them and the parts into which they divide each other. What do these measurements show?

12. A lot is in the form of a trapezoid $ABCD$. $AB = 100$ ft., $BC = 40$ ft., $CD = 60$ ft., and angles B and C are right angles. Draw the trapezoid on the scale of 20 ft. to 1 in.

13. Measure the opposite sides of a parallelogram. How do they compare in length?

14. Measure and find the sum of the angles in each of two trapeziums. What do you think the sum of the angles of a trapezium is?

15. Find at least one exercise in which each of the facts stated below is illustrated.

93. Principles concerning quadrilaterals. The following principles concerning quadrilaterals have been illustrated in the above exercises. The pupil should memorize these principles.

I. The sum of the angles of a quadrilateral is 360°.

II. In a parallelogram the opposite angles are equal and the adjacent angles are supplementary.

III. The opposite sides of a parallelogram are equal.

IV. The diagonals of a parallelogram bisect each other.

V. The diagonals of a rectangle are equal.

Exercise 86

1. In a parallelogram $ABCD$, $\angle A = 35° \ 20' \ 15''$. Find the size of each of the other angles.

2. The four angles of a quadrilateral are equal. How many degrees in each? What kind of quadrilateral is it?

3. Two boys lay out a tennis court. To test their work they measure the diagonals. One diagonal is 86 ft. and the other is 85 ft. Is the tennis court laid out properly? Give reasons for your answer.

4. A figure is known to be a parallelogram. It is found by measuring that one angle is 90°. What kind of parallelogram is it? Why?

5. One side of a square is s. What is the perimeter? State your answer as a formula, using p to represent the perimeter.

6. The perimeter of a square is p. What is the length of one side? State your answer as a formula.

7. Make a formula for finding the perimeter, p, of a parallelogram whose adjacent sides are a and b.

8. One angle of a parallelogram contains x degrees. How many degrees in each of the other angles?

9. Three angles of a parallelogram contain a degrees, b degrees, and a degrees, respectively. How many degrees in the fourth angle?

10. A trapezoid has two right angles. The third angle contains 47° 30′. Find the size of the fourth angle.

11. A quadrilateral has three equal sides. Is it necessarily a parallelogram? May it be a trapezoid? May it be a trapezium?

12. *Do the* diagonals of a parallelogram bisect the angles?

13. Make the following pattern.

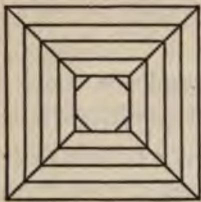


Fig. 83.

14. Make the following pattern.

Fig. 84.

15. Let each pupil copy a geometric design such as may be found in wall paper or linoleum patterns.

94. Polygons. A portion of a plane bounded by straight lines is called a **polygon.** The bounding lines are called the **sides** of the polygon.

Thus, AB, BC, CD, DE, and EA are the sides of the polygon $ABCDE$.

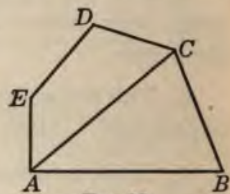


Fig. 85.

The points of intersection of the sides are called the **vertices** of the polygon. A, B, C, D, and E are the vertices of the polygon $ABCDE$.

A straight line joining two non-adjacent vertices of a polygon is called a **diagonal** of the polygon.

AC is a diagonal of the polygon $ABCDE$.

95. Kinds of polygons. Polygons are named according to the number of sides.

Number of Sides	Name	Number of Sides	Name
3	Triangle	7	Heptagon
4	Quadrilateral	8	Octagon
5	Pentagon	9	Nonagon
6	Hexagon	10	Decagon

Exercise 87

1. With a ruler construct an example of each of the polygons named in § 95.

2. Construct a pentagon and draw all of its diagonals.

3. Can you construct a polygon with only two sides? What is the least number of sides that a polygon may have? Can you name the greatest number of sides that a polygon may have?

4. What kinds of polygons are used in the woodwork in the schoolroom? What kind is most common?

5. What forms of tiles have you seen in floors, hearths, and mantels?

6. What kinds of polygons are the faces of a cube? The faces of the pyramid on page 115?

7. How many sides has a six-pointed star?

8. How many diagonals has a quadrilateral? A triangle? A hexagon?

9. Name as many kinds of polygons as you can that are shown in Figure 86.

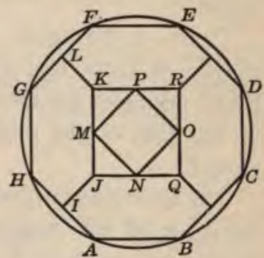


FIG. 86.

10. Make a copy of Figure 86. First draw a circle and divide it into 8 equal parts to locate the vertices of the octagon. Draw lines AF, BE, HC, and GD to get the sides of the square $JQRK$. I is the midpoint of AH. M is the midpoint of JK.

11. *Make a copy of the pattern given in Figure 87.*

FIG. 87.

12. Can you draw a polygon on the surface of a sphere? Give reasons for your answer.

13. Five children at play in a rectangular yard want to locate a point that is equally distant from the four corners. How can such a point be located?

96. The sum of the angles of a polygon. The diagonal of the quadrilateral, Figure 88, divides it into how many triangles? What is the sum of the angles of a triangle? Then what is the sum of the angles of the quadrilateral?

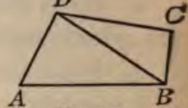


FIG. 88.

The diagonals of the pentagon, Figure 89, divide it into how many triangles? What is the sum of the angles of this pentagon?

Draw a hexagon. Draw the diagonals from one vertex. How many triangles are formed? What is the sum of the angles of a hexagon?

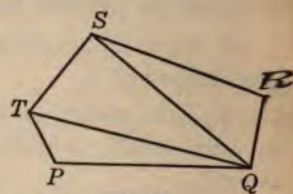


FIG. 89.

The above examples show that if we draw all the diagonals possible from one vertex of a polygon, the polygon is divided into a number of triangles. This number is two less than the number of sides of the polygon. The sum of the angles of each of these triangles is 180°. Hence if S is the sum of the angles of a polygon of n sides,

$$S = (n-2)180°.$$

Exercise 88

1. If the angles of a hexagon are equal, how many degrees are there in each angle?

2. How many degrees in the sum of the angles of an octagon?

3. Four of the angles of a pentagon are 40°, 95°, 120°, and 100°. How many degrees in the other angle?

4. Three of the angles of a hexagon are equal. The other angles are 50°, 65°, and 80°. How many degrees in each of the three equal angles?

5. If the angles of a polygon of 12 sides are all equal, how many degrees in each angle? If the angles of a polygon of n sides are all equal, how many degrees in each angle?

6. Copy and fill out the following table :

Name of polygon								
Number of sides								
Sum of the angles								

97. Regular polygons. A polygon whose sides are all equal and whose angles are all equal is called a **regular polygon.**


Fig. 90.


Fig. 91.

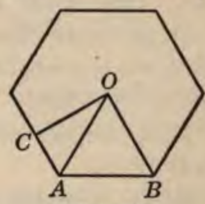


Fig. 92.

A line from the center to the vertex of a regular polygon is called the **radius** of the polygon.

Thus OA is the radius of the regular hexagon, Figure 92.

The angle between two successive radii is called the **central angle** of the regular polygon.

Angle AOB is the central angle of the regular hexagon.

The perpendicular from the center to one side is called the **apothem** of the regular polygon.

OC is the apothem of the regular hexagon.

Exercise 89

1. What other name has a regular triangle? What other name has a regular quadrilateral?

2. How many degrees in each angle of a regular triangle? Of a regular quadrilateral?

3. How many degrees in the sum of the angles of a regular hexagon? Of a regular octagon?

4. How many degrees in each of the angles of a regular polygon of 5 sides? Of 6 sides? Of 8 sides? Of 10 sides?

5. How many central angles has a regular hexagon? How many degrees in the sum of all of them? In one of them? Answer the same questions for a regular pentagon ; for a regular octagon ; for a regular decagon.

6. How many equilateral triangles can be placed so that each has one vertex at a given point P, all of the triangles lying in the same plane? How many squares can be so placed? How many pentagons? How many hexagons?

7. If you were choosing tiles to cover a floor, and were required to use a single form of regular polygon, except along the edges, what forms might be chosen?

8. Find the perimeter of a regular hexagon one side of which is 10 in.

9. Make a formula for finding the perimeter, p, of a regular polygon of n sides, each side being s inches long.

10. A polygon is said to be **inscribed in a circle** when the vertices of the polygon are on the circle. If a circle is divided into three or more equal parts, and consecutive points of division are joined by straight lines, a regular polygon is inscribed in the circle. Inscribe a square in a circle. See page 160.

11. Inscribe a regular hexagon in a circle. See page 127.

12. *Inscribe* an equilateral triangle in a circle by joining *alternate vertices* of a regular hexagon.

13. Inscribe a regular octagon in a circle. If you have the vertices of a square that is inscribed in a circle, how can you find the vertices of the regular octagon?

14. What is the sum of the angles of a regular quadrilateral? How many degrees in each angle? Answer the same questions for a regular pentagon; a regular octagon; a regular hexagon.

15. Make a table, similar to the table in exercise 6, page 175, for finding the number of degrees in an angle of a regular polygon of 3, 4, 5, 6, 7, 8, 9, and 10 sides.

16. What is the formula for finding the sum of the angles of a polygon of n sides? Make a formula for finding the number of degrees, A, in an angle of a regular polygon of n sides.

17. A circle may be divided into equal parts by drawing equal angles at the center. How many degrees must there be in each central angle to divide the circle into 3 equal parts? Into 4 equal parts? Into 5 equal parts? Into 8 equal parts?

18. Using the protractor to construct equal central angles, inscribe in a circle a regular pentagon; a regular octagon; a regular decagon.

19. Tell how the two 8-pointed star-polygons on page 163 may be made by using the protractor.

20. Using the protractor, divide a circle into ten equal parts. By joining the points of division in different ways make three different kinds of 10-pointed star-polygons.

Exercise 90. Review

1. Draw line segments a, b, and c and then construct a segment equal to $2a-b+c$.

2. *Show how to* obtain an angle by rotating a line.

3. Does changing the lengths of the sides of an angle change the size of the angle? Is an angle a portion of a plane surface?

4. If $\angle A = 8° 41' 10''$, find its complement and its supplement.

5. A man faces east. He turns to the left through 225°. In what direction is he then facing? Through how many degrees must he turn to the left to be facing east again?

6. Two angles of a triangle are equal and their sum is equal to the third angle. How many degrees in each angle?

7. Through how many degrees does a point on the circumference of a wheel turn while the wheel makes 4 revolutions?

8. Through how many degrees does a point on the surface of the earth turn in 1 hour? In 6 hours? In 1 minute? In 1 second?

9. Define parallel lines.

10. Define base, altitude, and diagonal of a parallelogram.

11. Define a regular polygon.

12. State the five principles concerning quadrilaterals given on page 171.

13. Construct the pattern in Figure 93.

FIG. 93.

FIG. 94.

14. Figure 94 is used as the basis of Gothic windows. Make it. D is the mid-point of AC. The center of the circle *is found by* drawing arcs with A and B as centers and BD *as a radius.*

15. Construct a square. Construct its radius and its apothem.

16. Inscribe a regular hexagon in a circle. Construct its radius and its apothem.

17. A number $6\frac{7}{8}$ hat is $6\frac{7}{8}$ in. in diameter. Find the length of the hat band.

18. The steamship *Carmania* sailing from Liverpool to New York was at noon on Aug. 27 in longitude 9° 19′ W.; Aug. 28, 20° 35′ W.; Aug. 29, 30° 52′ W.; Aug. 30, 39° 47′ W.; Aug. 31, 47° 55′ W.; Sept. 1, 57° 06′ W.; Sept. 2, 66° 09′ W. Find the number of degrees of longitude through which the ship sailed each day.

19. Do you know of any church windows that are decorated with designs which resemble Figure 94?

20. Measure the radii of the arcs in Figure 93. Find the sum of the lengths of all the arcs.

CHAPTER XI

AREAS

98. Comparison of rectangles.

1. Make a rectangle R with its length equal to a given line l and its width equal to a given line w. Make another rectangle, R', with its width w and its length $3\,l$. How many rectangles the size of R can be cut from rectangle R'?

2. Make another rectangle with length l and width $5\,w$. How many rectangles like R can be cut from this rectangle?

3. Construct a rectangle with length $2l$ and width $3\,w$. This new rectangle is how many times the rectangle R? Show by cutting the new rectangle.

4. A rectangle R has a length b and a width a, and rectangle R' has a length $4\,b$ and width $3\,a$. Rectangle R' is how many times rectangle R?

5. Draw a line a, 1 inch long. Construct a rectangle, R, with length $3\frac{1}{2}\,a$ and width $2\frac{1}{2}\,a$, and a rectangle, R', with length $\frac{1}{2}\,a$ and width $\frac{1}{4}\,a$. Show the number of times rectangle R contains rectangle R'.

6. Show by a drawing how many times a 2-inch square is contained in a 4-inch square; in a 6-inch square; in a 10-inch square.

7. What part of a square inch is a rectangle whose sides are $\frac{1}{2}$ in. and $\frac{1}{3}$ in.?

8. A rectangle 8 units long and $3\frac{1}{2}$ units wide contains the *square whose side* is 1 unit how many times?

9. If the length and width of a rectangle are measured by any unit of length, how can you find out how many times the rectangle contains a square whose side is 1 unit long?

10. The length of a rectangle is $\frac{4}{5}$ ft. and its width is $\frac{2}{3}$ ft. Its area is what part of a square foot? Show by a diagram.

SOLUTION. $AB = 1$ ft., $AE = \frac{2}{3}$ ft., and $AG = \frac{4}{5}$ ft.

Then, $ABCD$ is 1 sq. ft. and $AD = 1$ ft.

$ARST = \frac{1}{15}$ of $ABCD = \frac{1}{15}$ sq. ft.

$AEFG = 8 \times ARST = \frac{8}{15}$ sq. ft. $= \frac{2}{3}$ of $\frac{4}{5}$ of a square foot.

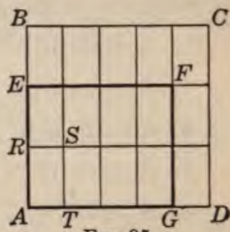


FIG. 95.

11. Make a rectangle whose length is $3\frac{3}{4}$ in., and its width $2\frac{2}{3}$ in. Show by a drawing that its area is $2\frac{2}{3} \times 3\frac{3}{4}$ sq. in.

Exercise 91

A yard of carpet means a strip one yard long but of any width which it happens to be made.

Wall paper is bought in rolls containing either 8 yards or 16 yards. It is usually 18 in. wide but is sometimes made other widths.

1. A certain room is 15 ft. long and 12 ft. wide. It is to be covered with carpet in strips 3 ft. wide, cut as long as necessary. If the strips are laid lengthwise of the room, how long must each strip be? How many strips will be needed? If laid crosswise of the room, how many strips will be needed? How long must each strip be?

2. A floor is 17 ft. long and 15 ft. wide. How many strips each a yard wide will be required to cover the floor if laid crosswise? If laid lengthwise? If the strips are laid crosswise what part of a strip must be cut off or turned under?

3. A floor is $16' \times 20'$. Draw it to the scale of 1 ft. to $\frac{1}{8}$ in. Show the number of strips of matting 3 ft. wide which are required to cover it, if laid lengthwise; if laid crosswise. In which case would there be the more waste?

Compute the number of yards of carpet required for the following rooms when the strips are laid lengthwise, also when laid crosswise, and decide which way requires the smaller amount of carpet. If in doubt, make drawings showing the carpet laid each way.

4. A room $10' \times 15'$, carpet a yard wide.

5. A room $13' \times 18'$, carpet a yard wide.

6. A room $7'\ 6'' \times 10'$, carpet $2'\ 6''$ wide.

7. A room $16' \times 18'\ 6''$, carpet a yard wide.

8. A hall $8' \times 22'\ 4''$, carpet $2'\ 9''$ wide.

9. A kitchen $11' \times 14'\ 6''$, with linoleum $6'$ wide.

10. A wall $7'\ 6''$ high and $16'$ long is to be papered. A roll of the paper used is $18''$ wide and contains 8 yd. A roll makes how many strips? Pieces of strips cannot be used. How many rolls are required?

11. The hearth before a fireplace is $6'$ long and $2'$ wide. It is laid with bricks each $8'' \times 4''$. If the bricks are laid lengthwise of the hearth, how many bricks are required along the long edge of the hearth? How many rows of bricks are required to cover the hearth? How many bricks are required to cover the hearth?

If laid crosswise of the hearth, how many bricks in each row? How many rows? How many bricks to cover the hearth?

12. A hearth $5' \times 1'\ 4\frac{1}{2}''$ is to be covered with tiles each $6'' \times 1\frac{1}{2}''$. How many tiles will be required to cover it?

13. If shingles have an average width of 4 in., how many are required for one row on a roof 22 ft. long?

Shingles are laid in rows overlapping each other. If three inches of the length of a shingle are not covered by *the overlapping* shingle, they are said to be laid 3 inches to *the weather*.

14. If the shingles are laid 4 in. to the weather, how many rows of shingles must be laid to cover a roof $12' \times 22'$? How many shingles? There is a double row at the bottom.

15. How many shingles are required for a roof $10'\ 6'' \times 28'$, laid $5''$ to the weather, the average width of a shingle being $4''$?

16. A certain roof requires 3560 shingles if they are laid 4 in. to the weather. How many does it require if they are laid 3 in. to the weather?

17. A field is 40 rd. $\times$ 80 rd. It is planted in corn in rows $3'\ 6''$ apart. The hills of corn are $3'$ apart in the rows. How many hills of corn are there in the field?

18. If tomatoes are planted in rows $5'$ apart with the plants $4'$ apart in the rows, how many plants are there in a field 30 rd. $\times$ 48 rd.?

19. How many paving bricks each $8'' \times 4'' \times 3''$ will be required to pave a street in front of a lot $40'$ wide, if the street is $30'$ wide and the bricks are laid with the face $3'' \times 8''$ up? If laid with the face $4'' \times 3''$ up?

99. Area of a rectangle. If the length and width of a rectangle are measured by the same unit of length, it is convenient to measure its area by a square whose side is that unit of length.

The pupil has seen that the number of unit squares is the product of the number of linear units in the base by the number of linear units in the altitude. This is what is meant by the formula

$A = ab$, and by the

Rule. *To find the area of a rectangle, multiply its base by its altitude.*

Exercise 92

1. A tennis court is 78 ft. long and 36 ft. wide. Find its area.

2. Four tennis courts are laid out side by side with a strip 10 ft. wide between adjacent courts, and a strip 15 ft. wide at each end. Find the area of the plot of ground used. Make a drawing of the four courts on the scale of 32 ft. to 1 in.

3. A baseball diamond is 90 ft. square. Find its area. This area is what part of an acre? Give the answer as a common fraction in its lowest terms, and as a decimal fraction correct to .001.

4. A owns a corner lot 60 ft. wide and 100 ft. deep. He puts down a concrete sidewalk 5 ft. wide along the two sides next to the street. The outside edge of the walk coincides with the edge of the lot. Find the cost of the sidewalk at 15 cents a square foot.

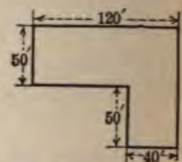


FIG. 96.

5. It costs $1.60 a square foot to lay a brick pavement. How much is that a mile for a pavement 9 ft. wide?

6. Find the area of this L-shaped lot. What is it worth at 12¢ a square foot?

7. A 15-inch square is how many times 15 sq. in.? Draw both on the scale of 5 in. to 1 in.

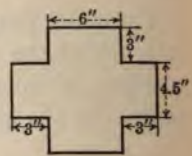


FIG. 97.

8. How many tiles 8 in. square will it take to cover a floor 14 ft. wide and 58 ft. long?

9. In making a box a boy cuts a piece of pasteboard in the shape of Figure 97. How many square inches of pasteboard are needed to make the box?

10. How much pasteboard is needed for a box 10 in. long, *$8\frac{1}{2}$ in. wide, and* $4\frac{1}{2}$ in. deep?

11. Find the area of the T-square given in Figure 98.

12. Make a formula for finding the area of the iron plate given in Figure 99. Compute the area if $a=12$ in. and $b=1\frac{3}{4}$ in.

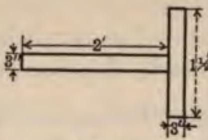


FIG. 98.

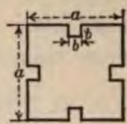


FIG. 99.

100. Area of a parallelogram. Construct a parallelogram $ABCD$ with its base, b, about 3 in. long, and its altitude a.

Construct the altitudes AM and DK.

Cut out the figure $ADCM$, then cut off the triangle DKC.

Show that the triangle DKC is the same size as triangle AMB by fitting the two together.

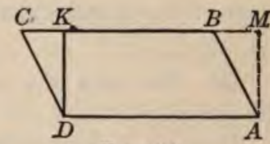


FIG. 100.

Rectangle $AMKD$ has, then, the same area as the parallelogram $ABCD$.

Rectangle $AMKD$ has the same base, b, and the same altitude, a, as the parallelogram $ABCD$.

The area of the rectangle $AMKD=ab$.

Therefore, the area of the parallelogram $ABCD=ab$.

Rule. *The area of a parallelogram equals the product of its base and its altitude.*

Exercise 93

1. Copy and complete this table, using the formula $A=ab$ to find the area, A, of each parallelogram.

$b=6'$	$15'$	$1.5'$	.02 mi.	$\frac{4}{5}$	$.2\frac{1}{3}$
$a=3'$	$6'$	$4'$	$\frac{1}{2}$ mi.	$1\frac{1}{4}$	$.0\frac{6}{7}$
$A=?$					

2. Find the area of a parallelogram whose altitude is 2% more than its base, which is 40 ft.

3. Find the area of a parallelogram whose base is 5% more than the altitude, the base being 315 ft. long.

4. A parallelogram has an area of 240 sq. in. Its altitude is 8 in. How long is its base?

5. A parallelogram has the same base as a rectangle whose area is 520 sq. ft. The altitude of the rectangle is 13 ft. and the altitude of the parallelogram is 20 ft. What is the area of the parallelogram?

101. The area of a triangle. Draw a triangle ABC, and its altitude BM.

Through B draw a line BH parallel to AC.

Through C draw line CH parallel to AB.

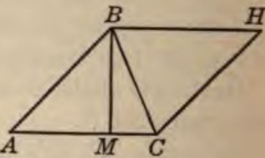


FIG. 101.

Cut out the parallelogram $ABHC$, then cut along the diagonal BC, thus making two triangles.

Show that these triangles are the same size by fitting them together.

If the area of the parallelogram is 6 sq. in., what is the area of triangle ABC?

The parallelogram and the triangle have the same base and the same altitude.

Since the area of the parallelogram is the product of its base and altitude, we have the

Rule. *The area of a triangle equals one-half the product of its base and altitude.*

State this rule as a formula, representing the area of the *triangle by T*, its base by b, and its altitude by a.

102. The area of a trapezoid. Draw a trapezoid $ABCD$. Prolong BC and AD, then lay off CE equal to AD and DF equal to BC. Draw EF, thus forming the parallelogram $ABEF$. Cut out this parallelogram, then cut along CD, and show that the two trapezoids are of the same size by fitting them together.

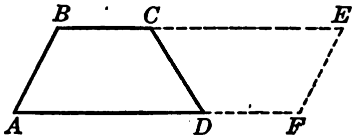


FIG. 102.

Trapezoid $ABCD$ is what part of the parallelogram $ABEF$?

Calling the lower base of the trapezoid, b, the upper base, b', and the altitude a, how long is the base of the parallelogram? Its altitude?

The area of the parallelogram is, therefore, $a(b+b')$.

Then the area of the trapezoid is $\frac{1}{2}a(b+b')$.

This is stated in the

Rule. *The area of a trapezoid is one-half the product of the altitude by the sum of the bases.*

The formula $T=\frac{1}{2}a(b+b')$ is more easily remembered than the rule.

Exercise 94

1. Use the formula $T=\frac{1}{2}ab$ to find the area, T, of the following triangles whose bases and altitudes are a and b:

a	12 in.	9 ft.	.6 in.	$\frac{3}{4}$ yd.	6 in.	1 rd.	1.08
b	7 in.	10 ft.	$\frac{1}{2}$ in.	$\frac{8}{9}$ yd.	2 ft.	20 in.	$.0\frac{1}{2}$
T	?	?	?	?	?	?	?

2. A railroad cut off a triangular piece of land from a farm. The longest side of the triangle was 24 rods and the distance of that side from the opposite corner was $8\frac{1}{4}$ rods. *How much* was this land worth at \$225 an acre?

3. If a half gallon of paint is required to paint a square (that is, a 10-foot square), how much is needed to paint three triangular gables each having a base of 15 ft. and an altitude of 8 ft.?

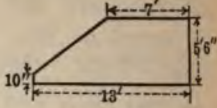


FIG. 103.

4. A section of a wall made by a stairway has the shape and dimensions shown in Figure 103. What is the cost of covering it with burlap at 25¢ a square yard?

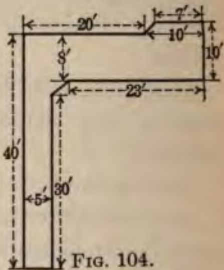


FIG. 104.

5. A concrete walk and a porch were laid at 20¢ a square foot. They were of the shape and dimensions given in Figure 104. Find the cost.

6. Construct a figure like Figure 105 and make a formula for finding its area. Find the area if $a=10''$ and $b=6''$.

7. Make a figure like Figure 106 and make a formula for finding its area. Find the area if $a=12''$ and $b=3''$.

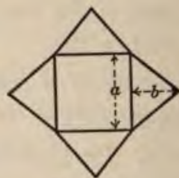


FIG. 105.

FIG. 106.

8. Find the number of yards of material 1 yd. wide that must be bought to make 100 pennants of the shape of Figure 107, if $a=8$ in. and $b=18$ in. Find the amount of waste if the material is bought all in one strip.

FIG. 107.

9. The 42 children in the seventh grade wish to make a pennant for each member of the class. The pennant is to have the form given in Figure 108.

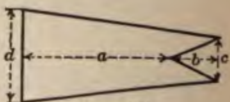


FIG. 108.

They think that the area of the pennant can be found *more easily* by first making a formula. Make this formula.

Find the area of a pennant if $a=18$ in., $b=6$ in., $c=6$ in., and $d=12$ in.

10. Figure 109 represents the end of an "I-beam," a form of steel beam used in building. Make a formula for finding the area of this figure. Find the area if $a=12$ in., $c=2$ in., and $b=4.5$ in.

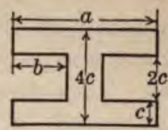


FIG. 109.

11. A concrete porch floor has the form of Figure 110. Make a formula for finding the area of this figure. Find the cost of laying this floor at 17¢ a square foot if $a=30$ ft., $b=20$ ft., $c=5\frac{1}{4}$ ft., and $d=7\frac{1}{2}$ ft.

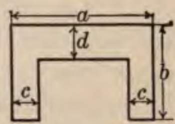


FIG. 110.

12. Manila paper comes in sheets 24 in. by 36 in., and costs 3¢ a sheet. The printer cuts it into sheets 12 in. by 18 in. What must he ask for 100 of these smaller sheets so as to gain 25% on the cost?

13. A garden is 30 ft. wide and 40 ft. long. The middle points of the opposite sides are joined by paths $2\frac{1}{2}$ ft. wide. Find the area that remains for cultivation.

14. *EFGH* and *FGLM*, Figure 111, are parallelograms. How do their areas compare? Give reasons for your answer.

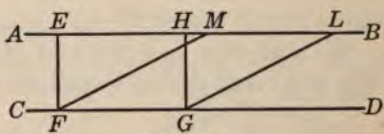


FIG. 111.

15. A ranchman has enough barbed wire to build 4 miles of fence. He wishes to inclose with it as large a rectangular field as possible. How long and how wide should he make the field?

16. A half-section of land is 160 rd. by 130 rd. It is divided into fields 40 rd. by 80 rd. by cross fences running parallel to the sides. Find the length of these cross fences.

17. The members of an arithmetic class wish to measure the school pond which is represented in Figure 112. They measure the line AD which is 265 ft. long. DC and AB are measured perpendicular to AD. $DC = 265$ ft., and $AB = 402$ ft. They decide that they can find the area of the pond as accurately as they desire if they regard EAF, GIH, LBM, NCO, and PDQ as triangles and subtract the sum of their areas from the area of $ABCD$. IJ is the altitude of GIH, BY is the altitude of LBM, and CX is the altitude of NCO. It is found by measuring that $EA = 98$ ft., and $AF = 82.5$ ft.; $GH = 130$ ft. and $IJ = 51$ ft.; $LM = 90$ ft. and $BY = 64.2$ ft.; $NO = 192$ ft. and $CX = 57.3$ ft.; $PD = 112$ ft. and $DQ = 73$ ft. Find the area of the pond in square feet; also in acres to the nearest .01 acre.

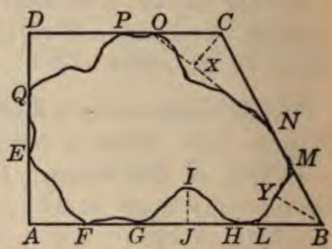


FIG. 112.

103. The area of a circle.

1. We first found the area of what kind of plane surface?

2. In finding the area of a parallelogram into what kind of figure was it changed?

We shall now find the area of a circle by first changing it to a form that closely resembles a figure whose area we know


FIG. 113.

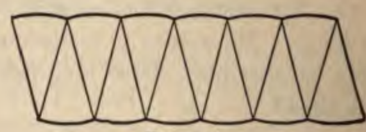


FIG. 114.

how to find. Cut a circle into 12 equal sectors by drawing *radii* as in Figure 113, and fit them together as in Figure 114.

3. What figure does Figure 114 resemble?

4. How does it differ from a parallelogram?

5. What line of the circle is the base of Figure 114? What line of the circle is its altitude?

6. If Figure 114 were exactly a parallelogram how could you find its area? What rule does this suggest for finding the area of a circle?

The answers to the above questions suggest the following rule which is proved in geometry:

Rule. *The area of a circle equals one-half the product of the radius and the circumference.*

This rule is more easily remembered in the formula

$$A=\frac{1}{2}rc,$$

in which A represents the area, r the radius, and c the circumference of the circle.

Since $c=2\pi r$, then $A=\frac{1}{2}r\times 2\pi r=\pi r^2$.

This formula $A=\pi r^2$ is the one most commonly used in finding the area of a circle. It is sometimes stated as the

Rule. *The area of a circle equals π times the square of the radius.*

Exercise 95

Find the areas of circles having the following radii. Use $\pi=3\frac{1}{7}$.

1. 7 in.	3. 5 ft.	5. .6 in.
2. 21 in.	4. $\frac{2}{3}$ in.	6. $27\frac{2}{3}$ ft.

In the following use $\pi=3.1416$.

7. 6 yd. 8. 25 ft. 9. 2 mi. 10. 20 rd.

11. Find the areas of circles with radii 1 in., 10 in., 50 in., 100 in., using both values of π. By how many square inches do the areas differ in each case? If we wish the answer correct to the nearest square inch, for which values of the radii may we use $\pi=3\frac{1}{7}$, and for which must we use $\pi=3.1416$?

12. A horse is tethered by a rope 20 ft. long. Over how large an area can he graze?

Find the areas of circles having the following diameters. Use $\pi = 3.1416$.

13. 12 ft.	**15.** 25 cm.	**17.** 2.046 ft.
14. .8 in.	**16.** 1.4 m.	**18.** $\frac{3}{5}$ in.

Find the areas of circles having the following circumferences. In the first three use $\pi = 3\frac{1}{7}$. In the next three use $\pi = 3.1416$.

19. 88 in.	**21.** 1 ft.	**23.** 25000 mi.
20. 37 cm.	**22.** 14137.2 Km.	**24.** 60 ft.

25. A circular race track is $\frac{1}{4}$ of a mile in circumference. What is its radius and its area?

26. What is the area of the largest circle that can be cut from a 10-inch square? The area of this circle is what per cent of the area of the square?

27. What part of Figure 115 represents the square on the diameter? What represents the square on the radius? The area of the circle is how many times the area of the square *AEOF*? The area of the circle is what part of the square *ABCD*?

Fig. 115.

28. In Figure 115 *AB* is 20 in. Find the area of the square on the diameter. Find the area of the square on the radius. Find the area of the circle. Check your answers to the last two questions in problem 27 by using the areas you have just found.

29. State in words the meaning of the formula $c = 2\pi r$.

30. State in words the meaning of the formula $A = \pi r^2$.

31. State in words the meaning of the formula $A = \frac{c}{2} r$.

Review Problems

Exercise 96. General Review

1. Mrs. Simpson has been buying flour in $3\frac{1}{2}$ lb. packages which cost 20¢ each. She decides to buy a bag of $24\frac{1}{2}$ lb. for \$1.25. Which is cheaper, and how much cheaper for $24\frac{1}{2}$ lb.?

2. One store advertises $3\frac{1}{2}$ lb. of sugar for 25¢, and another 5 lb. for 35¢. Which is cheaper?

3. There are 17 pupils in a cooking class and each needs $2\frac{1}{2}$ cups of water for a cooking lesson. How many quarts must there be in a kettle to supply all the class at one time, allowing 4 cups for a quart?

4. An automobile uses 1 qt. of oil every 100 miles and 1 gal. of gasoline every 18 miles. Oil costs 55¢ a gallon and gasoline 23¢ a gallon. Find the cost of oil and gasoline per mile, correct to .001¢. Find the cost of oil and gasoline for 100 miles; for 1000 miles.

5. A man buys a new automobile in the spring and drives it 5286 miles the first season. At the end of the season he wishes to compute his expenses. He estimates that the machine has depreciated \$300 in value. He has worn out 4 tires which cost \$23.40 each. He estimates that he has used 1 qt. of oil every 90 miles and 1 gal. of gasoline every 18 miles. Oil has cost him 58¢ a gallon and gasoline 22¢ a gallon. Repairs have cost \$9.60. Find the total expenses. Find the expense per mile. If the automobile carried on the average four passengers, was the cost of transportation of these four persons more or less expensive than traveling on the railroad at 2¢ a mile? How much?

6. A man owns $\frac{3}{5}$ of a stock of goods and sells $\frac{1}{2}$ of his share. What part of the stock does he sell? What part does he keep?

7. A owns $\frac{2}{5}$ of a farm. He sells $\frac{1}{4}$ of his share. What part of the farm does he still own? He still owns 120 acres. How many acres are there in the farm?

8. The radius of a circle is 6 ft. Find the area of a sector whose angle is 60°.

9. One angle of a triangular field is 120°. A goat grazing in the field is tied at this corner by a rope 75 ft. long. Over how many square feet of surface can the goat graze?

10. Carpenters estimate that in roofing it takes 800 shingles to cover 100 sq. ft. How many shingles are required for two sides of the roof of a barn 90 ft. long, the roof being 27 ft. from the comb to the eaves?

11. Masons estimate that it requires 21 bricks for each square foot of the face of a wall 12 in. thick, and 15 bricks for each square foot of the face of a wall 8 in. thick. How many bricks will be required for a wall 120 ft. long and 22 ft. high, 12 in. thick for the first 8 ft. of the height and 8 in. thick for the remainder?

12. Find the number of bricks required to make a wall 8 in. thick, 30 ft. long and 20 ft. high, after deducting for the openings for 6 windows each 3 ft. wide and 6 ft. high.

13. A machine stamps washers of the form of Figure 116 from a sheet of steel. How many washers of outside diameter 1 in. can be stamped from a sheet of steel 2 ft. by 3 ft.? If these washers are $\frac{1}{4}$ in. wide, what per cent of this sheet is wasted in cutting?

FIG. 116.

14. What is the total pressure on a piston head 18 in. in diameter if the pressure is 299 pounds to the square inch?

15. The railroad fare in Illinois was 2 cents a mile and was later increased $\frac{1}{5}$. To this was then added an 8% tax by the United States Government. What was then the fare from Chicago to Cairo, 364 miles? From Chicago to Springfield, 185 miles?

16. A certain traveling man traveled 8640 miles in one year on the railroads in Illinois. How much were his expenses increased by the increase in fare and the tax mentioned in the previous exercise?

17. The United States Supreme Court later decided that the increase mentioned in problem 15 should not be allowed and ordered the railroads to refund it. What was refunded a man who had paid the increase for 1578 mi. of travel?

18. When the fare between points within Illinois and between points within Indiana was 2 cents a mile, the fare from a point in one state to a point in the other was $2\frac{1}{2}$ cents a mile. A man traveling from East St. Louis, Illinois, to Indianapolis, Indiana, a distance of 249 miles, might buy a ticket for the whole distance at $2\frac{1}{2}$ cents a mile, or he might buy a ticket to Paris, Illinois, 158 miles, pay his fare to the conductor at 3 cents a mile from Paris to Terre Haute, Indiana, 19 miles, and buy a ticket from Terre Haute to Indianapolis, 72 miles. How much would be saved by the latter method?

19. When a heavy weight is suspended by a wire, the wire is stretched slightly. If a weight W is suspended by 10 ft. of a certain kind of copper wire the length l of the wire is given by the formula, $l = 10 + \frac{W}{11500}$. Find l if $W = 200$; if $W = 580$; if $W = 0$.

20. In measuring the side of a 4-foot square one boy obtained a result that was 3% too large and another boy obtained a result that was 3% too small. Each then used his result to find the area of the square. What was the per cent of error in the area in each case?

21. An error of 5% was made in measuring the radius of a circle which had a radius of 8 ft. That would cause what per cent of error in the area?

Exercise 97. Areas

1. Mr. Thomason buys a lot on which to build an elevator. It is bounded by two railroad tracks and a street as in Figure 117. He is to pay 25¢ a square foot for it. He computes the cost by measuring the line AC, which he finds to be 75 ft., and the distance from B to the line AC, which is 56 ft. How much must he pay for the lot?

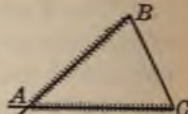


FIG. 117.

2. A hardwood floor was laid at 18¢ a square foot. The rooms had the shape and dimensions given in Figure 118. Find the cost of laying the floor.

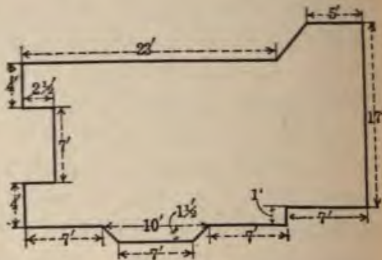


FIG. 118.

3. A cow is tied by a rope 42 ft. long to one corner of a shed which is 21 ft. square. Over how much area can she graze?

4. A piece of tin required for making a certain funnel has the shape and size indicated in Figure 119. How many square inches in it?

FIG. 119.

5. If the air pressure on the face of a kite is $1\frac{1}{2}$ lb. to the square foot, find the total pressure on the face of the kite shown in Figure 120.

6. At $1\frac{1}{2}$ lb. per square foot find the air pressure on the *face of the* kite of Figure *121*.

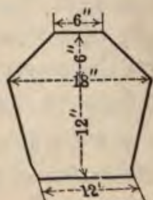


FIG. 120.

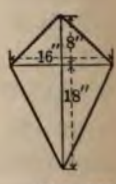


FIG. 121.

7. Find the area of the ring in Figure 122 if the radius of the inner circle is 10 in. and the radius of the outer circle 14 in. Obtain the same result by multiplying the width of the ring by the circumference of a circle midway between the inner and outer circles. Make a rule for finding the area of a circular ring.

FIG. 122.

8. A man agrees to cover a quarter-mile circular race track with cinders for 25¢ a square yard. The track to be covered is 20 ft. wide. The distance around the track is measured at the middle of the path to be covered. How much is he paid for the job?

Exercise 98. Problems of construction

1. Construct a triangle having two angles equal to two angles of a given triangle and the included side equal to one-half the included side of the given triangle.

2. Draw the bisector of the angle opposite the base of an isosceles triangle. Measure the angles the bisector makes with the base.

3. Construct a parallelogram. Bisect two of its consecutive angles. Measure the angles these two bisectors make with each other.

4. A line drawn perpendicular to a radius of a circle at its extremity in the circle is called a *tangent* to the circle. Draw a tangent to a given circle O at the point P on the circle.

5. Draw a circle with radius 1 in. tangent to the given line AB at the point N on the line AB.

6. Draw a circle having a given segment AB as diameter.

7. A funnel is made of a piece of tin of the shape given in Figure 123. The outer radius is $1\frac{1}{2}$ in., the inner radius $\frac{1}{2}$ in., and the piece cut out is one-fourth of the circle. Make a pattern for the funnel of the correct size.

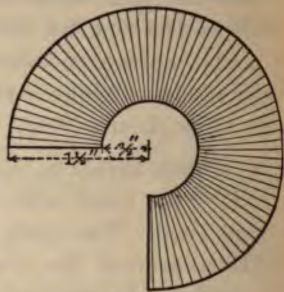


Fig. 123.

8. Draw a trapezoid having a given altitude, its lower base 3 in. and given acute angles at the ends of the lower base.

9. Construct Figure 105 having a and b of given length and a less than b.

10. Construct Figure 106 having a equal to 3 b.

11. Construct Figure 107 having b equal to 2 a.

Exercise 99. Finding heights and distances by drawing to scale

Engineers and surveyors must solve many problems in finding heights and distances. Certain of these problems can be solved easily and with sufficient accuracy for many purposes by drawing to scale.

1. Some boys wanted to find the distance across a river. They made the measurements given in Figure 124 and drew the figure to the scale of 40 feet to 1 in. They then measured the line AC in their drawing and found it to be $8\frac{1}{8}$ in. How far is it across the river? Draw the triangle and verify their result.

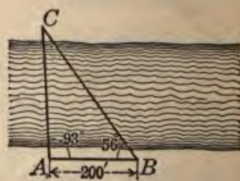


Fig. 124.

2. The boys wish to send up a kite to drop a message on the other *side of the river.* The kite string makes an angle of 48° *with the horizontal* line from A to C. In order to be safe

they wish to drop the message 60 ft. beyond C. How long must the kite string be, making no allowance for slack? Find by making a drawing to scale.

3. When the sun's rays make an angle of 61° with the horizontal a certain tree casts a shadow 42 ft. long. Find the height of the tree by drawing to scale.

4. The distance between two telephone poles on a straight road is 265 ft., Figure 125. When an observer looks at a certain house from the first pole the line of sight, BH, makes an angle of 64° with the road. When he looks at the same house from the second pole the line of sight, AH, makes an angle of 80° with the road. How far is the second pole from the house?

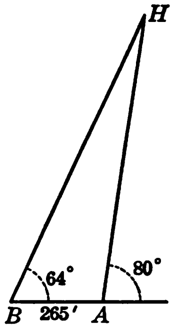


FIG. 125.

5. A boy holds a triangle as in Figure 126 and sights to the top of a tree. The distance from his eye to the tree is 50 ft. The angle of the triangle at his eye is 45°. The height of his eye from the ground is 5 ft. 1 in. Find the height of the tree.

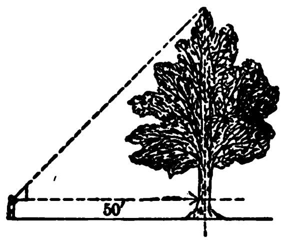


FIG. 126.

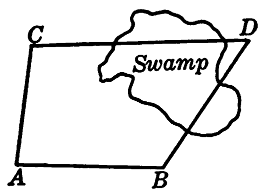


FIG. 127.

6. In Figure 127 $\angle A$ equals 85°, $\angle B$ equals 123°, and $\angle C$ equals 97°; AB equals 24 rd. and AC equals 20 rd. BD and CD cannot be measured because of a swamp. Make a drawing to scale and find these lines correct to the nearest rod. Also find $\angle D$.

Exercise 100. Investments

1. A man has \$1200 in a savings bank, which pays him 4% interest; \$2800 lent at 6% interest; \$300 lent at 7% interest; and \$3400 invested in a house and lot which rents for \$25 a month. What is his annual income from these investments?

2. Mr. King pays \$25 a month rent for the house in which he lives. He can buy the house for \$4500. If he does not buy the house, he can lend the money for 5% interest. He estimates that the taxes and insurance on the house will be \$95 a year. How much a year would he gain or lose by buying the house?

3. Many banks collect the interest in advance on the money that they lend. For example, if you were to borrow \$800 for 90 days at 7%, the bank would deduct the interest for 90 days at 7% and give you the remainder. How much would you receive? How much would the bank take out as interest? How much of the bank's money would you be using for the 90 days? What rate of interest would you be paying on the money used? Find the answer correct to .01%.

4. Answer similar questions if you were to borrow \$800 for 60 days at 7%.

5. Answer similar questions if you were to borrow \$800 for 30 days at 7%.

6. A savings bank pays 3% on deposits and lends these deposits at 6%. During 10 months of the year 80% of these deposits are lent; during the other two months of the year, because there is less demand for loans, only 50% are lent. How much does the bank make in one year on \$1000 deposits?

7. Find the profits of the savings bank in the previous exercise if it pays 4% on deposits and lends the deposits at 7%.

8. A real estate dealer pays \$6475 for a piece of property. He holds it six months and then sells it for \$6800. During the six months he receives \$200 rent and pays out \$68 for insurance and \$37.50 for repairs. How much does he make by the transaction? How much more does he make than if he had lent the \$6475 at 5%?

9. Before going to college a young man has an income of \$60 a month and spends \$500 a year. He takes a four-year college course which costs him \$2000. How much more money would he have had at the end of the four years if he had not gone to college, assuming that his income and expenses had remained the same? The first year after leaving college he makes \$110 a month and his expenses are \$600 a year. How much more does he save in a year than before going to college? If his income and expenses remain the same, after how many years will his savings amount to the same as if he had not gone to college?

Exercise 101. Problems of household economy

1. The United States Food Administration published on a certain day the following list which gives the maximum prices being paid for certain articles of food by retailers and the maximum price that consumers should pay:

Article	Price paid by Retailer	Price Consumer should pay
Cane sugar, per lb.	7.76¢	8.5¢
Flour, $\frac{1}{4}$ bbl.	\$2.92	\$3.18
Flour, 5-lb. bag	32¢	37¢
Butter, per lb.	45¢	50¢
Eggs, per dozen	45¢	50¢
Potatoes	\$2.35 for 100 lb.	43¢ for 15 lb.

(*a*) Find the per cent of profit made by the retailers on each article, correct to .1%.

(*b*) Find the amount of profit made on 100 lb. of sugar.

(*c*) Find the amount of profit on 100 lb. of butter.

(*d*) Find the amount of profit on 250 bu. of potatoes, 60 lb. to the bushel.

(*e*) If a housekeeper uses 3 bbl. of flour in a year, how much is saved by buying it $\frac{1}{4}$ bbl. at a time rather than in 5-lb. bags?

2. The average cost per week in Canada in six different years for rent, fuel, and lighting, and for 30 staple articles of food for a family of five persons is here given :

Year .	1910	1913	1914	1915	1916	1917
Cost .	\$12.76	\$14.00	\$14.28	\$13.82	\$14.75	\$16.74

The cost in each of the other years is what per cent higher than in 1910?

3. A wage earner in Canada received \$20 a week for 52 weeks in 1910. How much remained of the year's earnings after paying the expenses mentioned in the preceding problem? These expenses are what per cent of his earnings?

4. What must this wage earner receive a week in each of the years following 1910 if the expenses mentioned above are to remain the same per cent of his earnings as they were in 1910?

5. The following are suggested by students of household economy as proper distributions of incomes of various sizes in families of four or five persons :

Income	Food	Rent	Operating Expenses	Clothes	Other Expenses and Savings
\$500 to \$800	45%	15%	10%	10%	20%
\$800 to \$1000	30%	20%	10%	15%	25%
\$1000 to \$2000	25%	20%	15%	20%	20%
\$2000 to \$4000	25%	20%	15%	20%	20%

According to this table how much should be given to each item from a salary of $2500? How much from a salary of $1200? How much from a salary of $650?

6. A man receives a salary of $1800. If the price of food increases 35%, rent 10%, and clothing 25%, how much must his salary be increased so that he may have as much left as before for other expenses and for savings? Assume that he spends his salary according to the table given in the previous problem.

7. The following table shows the advance in prices in the Chicago market of certain articles of food from September 1, 1915, to September 1, 1917. There is also given an estimate of the amount of each used by a family of five in one year.

Article	Price in 1915	Price in 1917	Amount Used
Flour, per bbl.	$5.90	$11.40	3 bbl.
Eggs, per dozen	.24	$.37\frac{3}{4}$	150 doz.
Butter, per lb.	$.26\frac{1}{2}$	$.43\frac{1}{2}$	200 lb.
Potatoes, per bu.	.43	1.10	10 bu.

Find the per cent of increase in the cost of each of these articles. Find the total increase in the cost of the amounts used.

8. In the preceding exercise find the per cent of increase in the total cost of the food used. If this increase is the same for all the food used by a family, what is the increased cost of food in 1917 for a family which in 1915 had an income of $2500 and spent 25% of it for food?

9. The following dinner is planned for five people. The table gives the amount of each kind of food, and the per cent of protein, fat, and carbohydrate in each.

Roast beef Mashed potatoes Buttered asparagus
Lettuce salad French dressing
Strawberries Coffee

Food	Quantity	Protein	Fat	Carbohydrate
Beef	$2\frac{1}{2}$ lb.	13.9	21.2	
Potatoes	2 lb.	2.2	.1	18.4
Asparagus	1 lb.	1.8	.2	3.3
Lettuce	$\frac{1}{2}$ lb.	1.2	.3	2.9
Oil (Salad dressing) . .	3 oz.		100	
Strawberries	1 lb.	1.0	.6	7.4
Bread	6 oz.	9.2	1.3	53.1
Butter	3 oz.	1.0	85.0	
Milk	8 oz.	3.3	4.0	5.0
Sugar	6 oz.			100

Make a table showing the number of calories of protein, fat, and carbohydrate in each article of food. The number of calories produced by a given amount of protein, fat, and carbohydrate is given in exercise 1, p. 81.

10. Find the total number of calories each of protein, fat, and carbohydrate. Find the total number of calories furnished by the dinner.

11. Find the number of calories each of protein, fat, and carbohydrate in this meal for each of five persons.

TABLES

Length

12 inches	= 1 foot (ft.)
3 feet	= 1 yard (yd.)
$5\frac{1}{2}$ yards or $16\frac{1}{2}$ feet	= 1 rod (rd.)
320 rods, or 5280 ft.	= 1 mile (mi.)

Square Measure

144 square inches (sq. in.)	= 1 square foot (sq. ft.)
9 square feet	= 1 square yard (sq. yd.)
$30\frac{1}{4}$ square yards	= 1 square rod (sq. rd.)
160 square rods	= 1 acre (A.)
640 acres	= 1 square mile (sq. mi.)

Cubic Measure

1728 cubic inches (cu. in.)	= 1 cubic foot (cu. ft.)
27 cubic feet	= 1 cubic yard (cu. yd.)

Liquid Measure

4 gills (gi.)	= 1 pint (pt.)
2 pints	= 1 quart (qt.)
4 quarts	= 1 gallon (gal.)
$31\frac{1}{2}$ gallons	= 1 barrel (bbl.)
2 barrels	= 1 hogshead (hhd.)

Dry Measure

2 pints	= 1 quart (qt.)
8 quarts	= 1 peck (pk.)
4 pecks	= 1 bushel (bu.)

Weight

16 ounces (oz.)	= 1 pound (lb.)
2000 pounds	= 1 ton (T.)

Time

60 seconds (sec.)	= 1 minute (min.)
60 minutes	= 1 hour (hr.)
24 hours	= 1 day (da.)
7 days	= 1 week (wk.)
12 months (mo.)	= 1 year (yr.)
365 days	= 1 common year
366 days	= 1 leap year

Angles and Arcs

60 seconds (″) = 1 minute (′)
60 minutes = 1 degree (°)

Values

10 mills = 1 cent (¢ or ct.)
10 cents = 1 dime (d.)
10 dimes = 1 dollar ($)

Miscellaneous

1 cu. ft. of water weighs $62\frac{1}{2}$ lb.
1 qt. of milk weighs 2.18 lb.
$\pi = 3.1416$ or $3\frac{1}{7}$ nearly.
1 bu. shelled corn weighs 56 lb.
1 bu. ear corn weighs 70 lb.
1 bu. wheat weighs 60 lb.
1 bu. potatoes weighs 60 lb.
1 bu. rye weighs 56 lb.
1 bu. oats weighs 32 lb.
1 bu. apples weighs 44 to 50 lb.
1 barrel flour weighs 196 lb.
1 kilometer = about $\frac{5}{8}$ mi
1 meter = 39.37 in.
1 centimeter = about $\frac{2}{5}$ in.
1 liter = about 1 liquid quart.
1 kilogram = about $2\frac{1}{5}$ lb.

English Money

£1 (one pound) = 20 shillings = $4.87.

French Money

1 franc = 100 centimes = 19.3¢.

German Money

1 mark = 100 pfennigs = 23.8¢.

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